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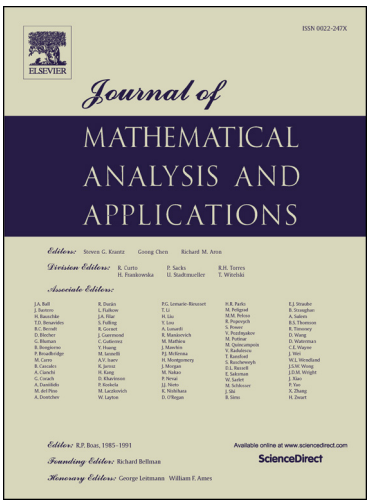

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LOCAL BISHOP-PHELPS-BOLLOBÁS PROPERTIES

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Abstract. In this paper we introduce some local versions of Bishop-Phelps-Bollobás type property for operators. That is, the function η which appears in their definitions depends not only on a given $\varepsilon > 0$, but also on either a fixed norm-one operator T or a fixed norm-one vector x. We investigate those properties and show differences between local and uniform versions.

1. Introduction

It is well-known that Bishop and Phelps proved in [5] the denseness of the set of all norm attaining functionals in X^*. They also asked if this result remains valid for bounded linear operators between any Banach spaces X and Y. Nevertheless, Lindenstrauss [23] proved that this is not true in general, by showing that there is a strictly convex Banach space $\mathcal{Z}$ such that the set of norm attaining bounded linear operators from c_0 into $\mathcal{Z}$ is not dense in the whole space of bounded linear operators from c_0 into $\mathcal{Z}$. Moreover, he started a systematic study of the conditions on the involved Banach spaces that guarantees an operator version of the Bishop-Phelps theorem. In 1970, Bollobás [6] improved the theorem of Bishop and Phelps by showing that, whenever we take a norm-one functional x^* and a norm-one point x satisfying that $x^*(x)$ is sufficiently close to 1, it is possible to find a new norm-one functional y^* and a new norm-one point y such that y^* attains its norm at y, y is close to x and y^* is close to x^*. This theorem is known nowadays as the Bishop-Phelps-Bollobás theorem. Motivated by Lindenstrauss' results, there has been an effort of many authors to study some geometric conditions of the Banach spaces X and Y in order to get a Bishop-Phelps-Bollobás type theorem for bounded linear operators from X into Y. The first one was the seminal work [2] due to M. Acosta, R. Aron, D. García and M. Maestre, where the Bishop-Phelps-Bollobás property for a pair of Banach spaces (X, Y) was introduced and studied. Essentially, a pair (X, Y) has the Bishop-Phelps-Bollobás property if a Bishop-Phelps-Bollobás type theorem holds for bounded linear operators from X into Y. They proved, among other results, that finite dimensional Banach spaces satisfy it and that, whenever Y has the Lindenstrauss property β, the pair (X, Y) has the Bishop-Phelps-Bollobás property for all Banach spaces X. A characterization of those Banach spaces Y such that the pair (ℓ_1, Y) has the Bishop-Phelps-Bollobás property was also given. After the mentioned article [2] in 2008, a lot of attention was given to this topic and many interesting problems related to this property were discussed. For more information the reader can refer, for example, to [1, 3, 4, 8, 9, 22].

To make the article entirely accessible, we present usual notations and necessary preliminaries. We work with Banach spaces X over the field $\mathbb{K}$, which can be either the set of real numbers $\mathbb{R}$, or the set of complex numbers $\mathbb{C}$. We denote by S_X, B_X and X^* the unit sphere, the unit ball and the topological dual of X, respectively, and by $\mathcal{L}(X, Y)$ the set of all bounded linear operators from X into Y. We say

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