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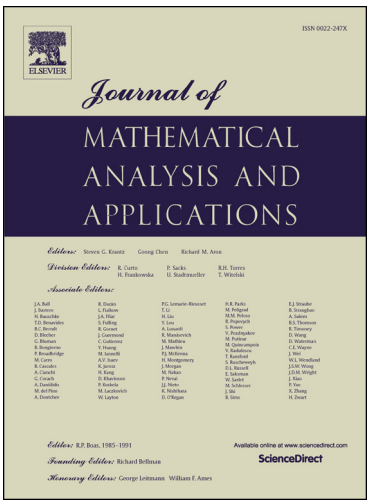

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ANALYSIS ON A COUPLED PARABOLIC SYSTEM WITH FREE BOUNDARY

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Abstract. The purpose of this paper is to investigate a parabolic problem with coupled reaction terms and free boundary, including the existence, uniqueness, regularity and long-time behavior of positive solutions. Firstly, by the contraction mapping theorem, we establish the (local) existence and uniqueness of positive solutions. Then, we prove the solution blows up in finite time with large initial data by comparison principle, and give more details for these large initial data by exhibiting an energy condition. The simultaneous blow-up result of the maximum of u and v is obtained, and the blow-up set of blow-up solutions is a compact subset of $[0, h_0]$. Furthermore, there is a global and fast solution, which decays uniformly at an exponential rate if the initial datum is small, while there is a global and slow solution provided that the initial datum is suitably large. Finally, for initial data $\sigma(\varphi(x), \psi(x))$, we obtain a trichotomy conclusion by considering the size of parameter σ.

1. Introduction

In this paper, we consider the following one-phase Stefan problem:

$$
\begin{cases}
u_t - du_{xx} = -\lambda_1 u + \alpha_1 u^3 + \beta u v^2, & t > 0, \ \ 0 < x < h(t), \\
v_t - dv_{xx} = -\lambda_2 v + \alpha_2 v^3 + \beta u^2 v, & t > 0, \ \ 0 < x < h(t), \\
u_x(t,0) = u(t,h(t)) = 0, \ \ v_x(t,0) = v(t,h(t)) = 0, & t > 0, \\
h'(t) = -\mu(u_x(t,h(t)) + \rho v_x(t,h(t))), & t > 0, \\
h(0) = h_0, \ \ u(0,x) = u_0(x), \ v(0,x) = v_0(x), & 0 \le x \le h_0,
\end{cases} \tag{1.1}
$$

where $x = h(t)$ is the free boundary to be determined, $h_0 > 0$, d, λ_i, $\alpha_i (i = 1, 2)$, β are positive constants.

As we know, the free boundary problems have been used to describe different types of mathematical models. Let us recall some work about the existence and blow-up results to the reaction-diffusion equations or systems with free boundaries. Recently, Ghidouche, Souplet and Tarzia [12] considered the Stefan problem

$$
\begin{cases}
u_t - u_{xx} = u^p, & t > 0, \ 0 < x < h(t), \\
u(t,h(t)) = u_x(t,0) = 0, & t > 0, \\
h'(t) = -\mu u_x(t,h(t)), & t > 0, \\
u(0,x) = u_0(x) \ge 0, & 0 \le x \le h_0.
\end{cases} \tag{1.2}
$$

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