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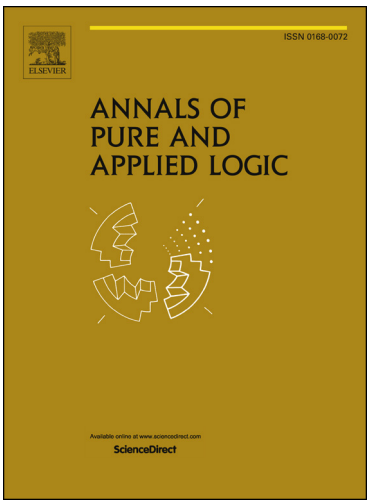

First order logic without equality on relativized semantics

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FIRST ORDER LOGIC WITHOUT EQUALITY ON RELATIVIZED SEMANTICS

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Abstract. Let $\alpha \geq 2$ be any ordinal. We consider the class Drs_α of relativized diagonal free set algebras of dimension α. With same technique, we prove several important results concerning this class. Among these results, we prove that almost all free algebras of Drs_α are atomless, and none of these free algebras contains zero-dimensional elements other than zero and top element. The class Drs_α corresponds to first order logic, without equality symbol, with α-many variables and on relativized semantics. Hence, in this variation of first order logic, there is no finitely axiomatizable, complete and consistent theory.

1. Introduction

In the middle of the twentieth century, A. Tarski introduced and initiated the investigation of cylindric algebras and relation algebras. These algebras are Boolean algebras with extra additive, closure and complemented operators. The theories of these algebras are directly related to the development of some versions of quantifier logics, e.g., classical first order logic. These theories (and the theories of the related structures) have found interesting realizations and applications in mathematics, computer science, philosophy and logic, c.f., e.g., [15], [16], [17], [18] and [37].

An important notion in the theories of these algebras is the notion of representable algebras. These algebras can be conceived as expansions of Boolean set algebras whose elements are unary relations to algebras whose elements are relations of higher ranks. The question whether every abstract algebra is isomorphic to a representable algebra is the algebraic equivalent of the completeness theorem for the corresponding logic. Representable algebras represent the semantics of the corresponding logic, while abstract algebras correspond to its syntactical side.

One can find well motivated appropriate notions of representable structures by first locating them while giving up classical semantical prejudices. It is hard to give a precise mathematical underpinning to such intuitions. What really counts at the end is a completeness theorem stating a natural fit between chosen intuitive concrete-enough, but perhaps not excessively concrete, semantics and well behaved, hopefully recursive, axiomatization. Gödel's completeness theorem ties just one choice of logical validity in standard set theoretic modeling.

The classical concrete algebras are cylindric set algebras defined by A. Tarski, these are algebras of sets of sequences in which the top element is a square of the form ${}^\alpha U$, where U is a non-empty set and α is the dimension. Other concrete algebras can be the *relativized* versions of cylindric set algebras. The top element of a relativized set algebra is arbitrary subset $V \subseteq {}^\alpha U$ with operations

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