

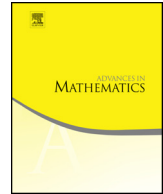


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The homotopy theory of type theories

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ABSTRACT

We construct a left semi-model structure on the category of intensional type theories (precisely, on $\mathbf{CxlCat}_{\mathbf{Id},1,\Sigma(\cdot,\Pi_{\text{ext}})}).$ This presents an ∞ -category of such type theories; we show moreover that there is an ∞ -functor \mathbf{Cl}_∞ from there to the ∞ -category of suitably structured quasi-categories.

This allows a precise formulation of the conjectures that intensional type theory gives internal languages for higher categories, and provides a framework and toolbox for further progress on these conjectures.

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1. Introduction

Homotopy Type Theory (HoTT) arises from the discovery that the logical system of dependent type theory can be naturally interpreted in homotopy-theoretic settings, and provides a rich language for such settings, thanks largely to its richer treatment of equality compared to first-order logic.

Classically, equality carries no information beyond its truth value; but in type theory, objects can be equal/identifiable in a variety of ways, so a type of equalities may be a

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non-trivial type in its own right, analogous to the path space of a topological space. This forms the basis of “synthetic homotopy theory”, a major direction within HoTT: the development of homotopy-theoretic constructions and theorems, entirely elementarily within type theory.

Of course, one wants to know that these can be interpreted in a good range of established settings that a homotopy theorist might care about. Conversely, one may hope that all homotopy-theoretic concepts (in some sense) can be translated into type theory. Various results in these directions have been given — some proven, some conjectured, some only informally sketched (see e.g. [14,24–27,17]).

Such hopes have been summarized as the idea that HoTT should be the **internal language of ∞ -categories**. Precisely, by analogy with established “internal languages” in settings such as topos theory, this should mean a single master statement subsuming the above results: the existence of a suitable equivalence between some (higher) categories of type theories and ∞ -categories.

The first contribution of the present paper is a framework for formulating such a claim precisely. We do so by assembling type theories into a higher category, and giving a functor Cl_∞ from this to a higher category of suitably structured quasicategories. The internal language conjecture can then be stated as: Cl_∞ is an equivalence of higher categories.

The other main contribution is a left semi-model structure on the category of type theories. This gives a tractable and explicit presentation of the higher category thereof, which we hope will provide a solid base for further progress on the conjectures.

In a little more detail: we work with “type theories” as contextual categories or categories with attributes (CwA’s), keeping our results independent of the correspondence between these and syntactically presented theories. We assume Id -, Σ -, and unit types throughout; we consider also the extension to Π -types.

Two technical tools of the paper may be of independent interest. One (small, but useful and to our knowledge new) is a notion of equivalence between arbitrary objects of a CwA. The other is the construction of the CwA of span-equivalences in a given CwA, a powerful tool for constructing equivalences between CwA’s.

In Section 5, we make use of some results from our forthcoming article [19], currently in preparation. However, those results may be treated as black boxes; the present paper can be read as essentially self-contained.

During the preparation of this paper, we learned that Valery Isaev has independently given a similar construction in [12], defining a (full) model structure on a slightly different category of type theories (assuming an interval type, instead of Martin-Löf identity types).

2. Background

In this section, we review the necessary background on categorical models of type theory. We recall the definition of a contextual category and introduce the notation for

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