ARTICLE IN PRESS

Composite Structures xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Stationary random response of non-viscously damped polymer matrix composite structure systems

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A R T I C L E I N F O

Keywords:
Stationary random response
Non-viscously damped composite structure
Pseudo excitation method
Direct frequency response method
Iterative method

A B S T R A C T

The polymer materials are widely used as the matrix of composite structures in the automobile, aircraft, building, bridge, railway, ship, turbine, and appliance industries. The polymer materials with long-chain molecules exhibit non-viscous damping behaviour. Non-viscous damped composite structure systems in which the damping forces depend on the past history of velocities via convolution integrals over some kernel functions have been raised in many different subjects. The developed analysis methods of dynamic response for such structure systems are almost limited to the deterministic time-history excitation. There is little research report on the random response of such non-viscous damped systems. The author has previously developed the non-viscous damping modelling method for polymer matrix composite structures (Liu, to be published). The goal of this paper is focus on developing two methods, i.e., direct frequency response method and iterative method using real modes, to obtain the power spectral density function (PSDF) of non-viscously damped polymer matrix composite structure systems subjected to stationary stochastic excitation. First, the pseudo excitation method converts the stationary stochastic excitation problem into harmonic excitation problem. Second, the direct frequency response method is derived and proven to get the analytical solution of PSDF. Third, the iterative method using real modes to obtain PSDF matrix is developed based on a harmonic response method. The computational procedure of the iterative method is given in detail. Finally, the random response analyses of two non-viscously damped structure systems, subjected to stationary random excitation, are demonstrated. The results indicate the two methods can achieve the exact solution of PSDF matrix of non-viscously damped structure systems. The iterative method using real modes is more efficient than direct frequency response method.

1. Introduction

The non-viscous damping has risen in many engineering fields [1–3]. For example, the polymer materials with long-chain molecules exhibit viscoelastic damping behaviour. The polymer materials are widely used as the matrix of composite structures in the automobile, aircraft, building, bridge, railway, ship, turbine, and appliance industries [4]. The viscoelastic damping material is frequency-dependent or strain rate related. Boltzmann [5] first developed Boltzman's Superposition Principle (a convolution integral expression) to describe the stress-strain relationship of the linear isotropic viscoelastic damping material. In the recent decades, many non-viscous damping models have been developed to deal with the non-viscous damping. Biot [6,7] damping model, Buhariwala [8] damping model, Bagley and Bagley and Torvik [9], Torvik and Bagley [10] damping models, Golla and Hughes [11], McTavis and Hughes [12] damping model, Anelastic Displacement Field [13,14] damping model, Gaussian damping model and exponential damping models [15]. The non-viscous damping models, in which the damping forces depend on the past history of velocities via convolution integrals over some kernel functions, has been pointed out to be the most general damping model within the linear range by Woodhouse [16]. Adhikari [17] has summarized what type of structure systems may have non-viscous damping.

The dynamic behaviour of the non-viscously damped structure systems, including identification of non-viscous damping model parameters [15,18,19], modal analysis [20–22], dynamic response analysis [23–25], have been widely investigated in the recent decades. The developed analysis methods of dynamic response for the non-viscously damped structure systems are almost limited to the deterministic time-history excitation. There is little research report on the random response of the non-viscously damped structure systems. Adhikari [20] calculated the power spectral density function (PSDF) of a 3-DOF non-viscous damping system, subjected to a band-limited Gaussian white noise, using directly a kernel formula of random vibration theory, i.e.,

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http://dx.doi.org/10.1016/j.compstruct.2017.09.101
Received 22 September 2017; Accepted 28 September 2017

$\boldsymbol{S}_x(\omega) = \boldsymbol{H}^*(\mathrm{i}\omega)\boldsymbol{S}_f(\omega)\boldsymbol{H}^T(\mathrm{i}\omega)$, where $\boldsymbol{S}_f(\omega)$ and $\boldsymbol{S}_x(\omega)$ are input and output PSDF matrices, $\boldsymbol{H}^*(\mathrm{i}\omega)$ and $\boldsymbol{H}^T(\mathrm{i}\omega)$ are the complex conjugate and transpose of the frequency response function matrix, respectively. The kernel formula has been proved not efficient to compute the PSDF of viscous damping structure systems with large-scale degree of freedom. The computational method combining the kernel formula with the modal superposition method may improve the efficiency [26]. However, to solve the complex modes of the non-viscous damping structure systems is very much time-consuming because the frequency-dependent damping matrix causes the nonlinear eigenproblem [27,28]. Therefore, the method combining the kernel formula with the modal superposition method may not improve the efficiency for non-viscous damping structure systems with large-scale degree of freedom.

The author has previously developed the non-viscous damping modelling method for polymer matrix composite structures [29]. The aim of this paper is to develop two computational methods to obtain the power spectral density function (PSDF) matrix of non-viscously damped polymer matrix composite structure systems subjected to stationary stochastic excitation. First, the stationary stochastic response problem is converted into harmonic response problem by using the pseudo excitation method. Then any methods for calculating harmonic response can be employed to find the PSDF matrix of the non-viscously damped structure systems. Second, the direct frequency response method is derived and proven to get the analytical solution of PSDF. Third, the iterative method using real modes to obtain the matrix of PSDF is developed based on a harmonic response method. The computational procedure of the iterative method is given in detail. Finally, the random response analyses of the single story frame model with non-viscous damper (SDOF question) and multiple-story frame model with non-viscous dampers (MDOF question), subjected to stationary random ground motion, are demonstrated. The accuracy and efficiency of the two methods are also discussed.

2. Stationary response analysis

The governing equations of the non-viscously damped structure systems subjected to the stationary random excitation with zero initial conditions can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \int_0^t \boldsymbol{G}(t-\tau)\dot{\boldsymbol{x}}(\tau)\mathrm{d}\tau + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{M}$ is mass matrix, $\boldsymbol{K}$ is stiffness matrix, $\boldsymbol{G}(t)$ is a symmetric matrix of the damping kernel functions of the non-viscously damped structure systems. $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are the displacement vector, velocity vector and acceleration vector, respectively. $\boldsymbol{f}(t)$ is a stationary random excitation vector.

The pseudo excitation method can be applied to any linear systems [30–32]. Therefore, it is also can be employed to non-viscously damped linear structure systems. The pseudo excitations of the stationary random excitation vector $\boldsymbol{f}(t)$ can be expressed as

$$\tilde{\boldsymbol{f}}(t) = \sqrt{\boldsymbol{S}_f(\omega)}e^{\mathrm{i}\omega t} \tag{2}$$

where $\boldsymbol{S}_f(\omega)$ are the known power spectrum vector of the stationary random process vector $\boldsymbol{f}(t)$.

If the non-viscously damped linear structure systems are subjected to the pseudo excitations $\tilde{\boldsymbol{f}}(t)$, the governing Eq. (1) become

$$\boldsymbol{M}\ddot{\tilde{\boldsymbol{x}}}(t) + \int_0^t \boldsymbol{G}(t-\tau)\dot{\tilde{\boldsymbol{x}}}(\tau)\mathrm{d}\tau + \boldsymbol{K}\tilde{\boldsymbol{x}}(t) = \tilde{\boldsymbol{f}}(t) \tag{3}$$

where $\tilde{\boldsymbol{x}}(t)$, $\dot{\tilde{\boldsymbol{x}}}(t)$ and $\ddot{\tilde{\boldsymbol{x}}}(t)$ are called the pseudo displacement response vector, velocity response vector and acceleration response vector, respectively, of the non-viscously damped linear structure systems subjected to the pseudo excitations $\tilde{\boldsymbol{f}}(t)$.

Substituting Eq. (2) into Eq. (3), we have

$$\boldsymbol{M}\ddot{\tilde{\boldsymbol{x}}}(t) + \int_0^t \boldsymbol{G}(t-\tau)\dot{\tilde{\boldsymbol{x}}}(\tau)\mathrm{d}\tau + \boldsymbol{K}\tilde{\boldsymbol{x}}(t) = \sqrt{\boldsymbol{S}_f(\omega)}e^{\mathrm{i}\omega t} \tag{4}$$

The aforementioned Eq. (4) indicates that the pseudo responses are the harmonic responses of the non-viscously damped linear structure systems subjected to harmonic excitations, i.e., $\sqrt{\boldsymbol{S}_f(\omega)}e^{\mathrm{i}\omega t}$. Any methods (direct frequency response method, complex mode superposition method and iterative method) to solve the harmonic responses of the non-viscously damped structure systems can be used to achieve the pseudo responses $\tilde{\boldsymbol{x}}(t)$. However, the complex mode analysis of the non-viscously damped structure system is very much time-consuming because the frequency-dependent damping matrix brings the nonlinear eigenproblem. Therefore, direct frequency response method and iterative method are employed to calculate the pseudo responses in this paper.

When the pseudo responses are obtained, according to the pseudo excitation method, the PSDF matrix of the displacements is

$$\boldsymbol{S}_x(\omega) = \tilde{\boldsymbol{x}}^*(t)\cdot[\tilde{\boldsymbol{x}}(t)]^T \tag{5}$$

where the superscript '$*$' is a complex conjugate and the superscript 'T' is transpose operation.

3. Direct frequency response method for PSDF matrix of the displacements

For a linear system subjected to the harmonic excitation, i.e., Eq. (4), the responses can be assumed as

$$\tilde{\boldsymbol{x}}(t) = \tilde{\boldsymbol{X}}(\mathrm{i}\omega)e^{\mathrm{i}\omega t} \tag{6}$$

Substituting Eq. (6) into Eq. (4) and using the zero initial conditions, we have

$$[-\omega^2\boldsymbol{M} + \mathrm{i}\omega\boldsymbol{G}(\mathrm{i}\omega) + \boldsymbol{K}]\tilde{\boldsymbol{X}}(\mathrm{i}\omega) = \sqrt{\boldsymbol{S}_f(\omega)} \tag{7}$$

where $\boldsymbol{G}(\mathrm{i}\omega) = L[\boldsymbol{G}(\mathrm{t})]$ with Laplace variable $s = \mathrm{i}\omega$. We rewrite Eq. (7) as

$$\boldsymbol{D}(\mathrm{i}\omega)\tilde{\boldsymbol{X}}(\mathrm{i}\omega) = \sqrt{\boldsymbol{S}_f(\omega)} \tag{8}$$

or

$$\tilde{\boldsymbol{X}}(\mathrm{i}\omega) = \boldsymbol{H}(\mathrm{i}\omega)\sqrt{\boldsymbol{S}_f(\omega)} \tag{9}$$

where $\boldsymbol{D}(\mathrm{i}\omega) = [-\omega^2\boldsymbol{M} + \mathrm{i}\omega\boldsymbol{G}(\mathrm{i}\omega) + \boldsymbol{K}]$ is called dynamic stiffness matrix of the non-viscously damped structure systems. $\boldsymbol{H}(\mathrm{i}\omega) = [-\omega^2\boldsymbol{M} + \mathrm{i}\omega\boldsymbol{G}(\mathrm{i}\omega) + \boldsymbol{K}]^{-1}$ is called frequency response function matrix (also called dynamic compliance matrix) of the non-viscously damped structure systems.

For a problem with one or two degree of freedom, we can obtain the analytical solution of Eq. (7). Therefore, the analytical solution of PSDF matrix, i.e. Eq. (5), can be achieved. For a problem with the large-scale degree of freedom, it is difficult to find the analytical expression of $\tilde{\boldsymbol{X}}(\mathrm{i}\omega)$ in Eq. (7). However, the exact solution of $\tilde{\boldsymbol{X}}(\mathrm{i}\omega)$ at any frequency points can still be obtained by using inversion of the matrix $\boldsymbol{D}(\mathrm{i}\omega)$ at each interesting frequency point.

When $\tilde{\boldsymbol{X}}(\mathrm{i}\omega)$ is obtained, we substitute Eq. (6) into Eq. (5) to have

$$\boldsymbol{S}_x(\omega) = [\tilde{\boldsymbol{X}}(\mathrm{i}\omega)e^{\mathrm{i}\omega t}]^*[\tilde{\boldsymbol{X}}(\mathrm{i}\omega)e^{\mathrm{i}\omega t}]^T = \tilde{\boldsymbol{X}}^*(\mathrm{i}\omega)\tilde{\boldsymbol{X}}^T(\mathrm{i}\omega) \tag{10}$$

In addition, the PSDF matrix obtained using Eq. (5) can be proved to be the analytical solution. Substituting Eq. (9) into Eq. (10), we obtain

$$\begin{aligned}\boldsymbol{S}_x(\omega) &= \tilde{\boldsymbol{X}}^*(\mathrm{i}\omega)\tilde{\boldsymbol{X}}^T(\mathrm{i}\omega)\\ &= (\boldsymbol{H}(\mathrm{i}\omega)\sqrt{\boldsymbol{S}_f(\omega)})^*\cdot[\mathrm{H}(\mathrm{i}\omega)\sqrt{\boldsymbol{S}_f(\omega)}]^T\\ &= \boldsymbol{H}^*(\mathrm{i}\omega)\sqrt{\boldsymbol{S}_f(\omega)}\cdot[\sqrt{\boldsymbol{S}_f(\omega)}]^T\boldsymbol{H}^T(\mathrm{i}\omega)\\ &= \boldsymbol{H}^*(\mathrm{i}\omega)\boldsymbol{S}_f(\omega)\mathrm{H}^T(\mathrm{i}\omega)\end{aligned} \tag{11}$$

Now the PSDF matrix obtained using Eq. (5) is proved to be the same as the analytical solution (kernel formula) of the random vibration theory.