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Methods for determining the plastic work in bending and impact of selected factors on its value

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ABSTRACT

The paper describes various methods which may be used in determining the deformation energy in the plastic region of bent wood. The present work focuses on various definitions of the plastic work of wood as well as the influence of various factors on its value. Approximated plastic work and total plastic work were determined for beech (*Fagus Sylvatica* L.) and aspen (*Populus Tremula* L.) wood. We took into consideration the following factors: wood species (**WS**) (*Fagus Sylvatica* L. and *Populus Tremula* L.), material thickness (**MT**) (4, 6, 10, and 18 mm), degree of densification (**DOD**) (10% and 20% of the original thickness), and the number of stressing cycles (**NC**) (0 versus 10,000). The study brings a new quantitative expression of an approximated plastic work "$\mathbf{A_W}$", and total plastic work "$\mathbf{B_W}$" and the values of approximation error "**ΔW**", which are made due the applied approximation.

1. Introduction

Each change of a shape of the bent wood results from work which mainly depends on the affecting force and achieved deflection (Fig. 1).

The hatched area in Fig. 1 characterizes the total work used for the deformation of the body [1]. The size of this area depends not only on the maximum loading "F_P" and deflection at the moment of damage but also on a shape of the curve, wood moisture and density.

The area size declines with the increasing moisture and decreasing wood density for one specific wood species, or groups of wood species (soft- and hardwoods) [2].

The calculation of work used for deformation of the body is demonstrated in the relation (1):

$$A_C = \varphi.\ F_P.\ Y_P \tag{1}$$

where A_C is the total work (J), F_P (N) is the force at the modulus of rapture "MOR", Y_P (mm) is the deflection at the MOR, and φ the degree of the area filling. In literature the value of 0.80 can be found. As a result of wood defects, wood moisture or other reduction it goes down to 0.50 [3].

By dividing the total work at Impact Bending Strength "A_D" through the transverse area of the body we obtain the value which is used as the characteristic of wood toughness

$$H = A_D/S \tag{2}$$

where H is wood toughness (N), A_D is the total work used for damaging the body at Impact Bending Strength (J), S is the transverse area of the bent body (mm^2). The term "toughness" is also used for the product of the elasticity modulus and moment of inertion.

When deformation in the elastic area takes place, external forces generate work [3]. Internal forces generate identical work called deformation work. In the deformed body, potential energy accumulates and after unloading the body goes back to its original state [4]. In linear part it is not a problem to determine work in the elastic area. To determine it we can use the equation for the calculation of elastic potential which we used in the publication [5]

$$\overline{P_E} = \frac{3}{2}\frac{Fy_E}{bhl_o} \tag{3}$$

If the force F is expressed in Newtons and dimensions (b, h, l_0) and the deflection y_E is expressed in mm, then the resulting unit of P_E is MPa, i.e. MJ/m^3.

If we multiply potential "P_E" by the volume of the stressed body, we obtain the value of elastic work "A_P" (4)

$$A_P = P_E.\ b.\ h.\ l_0 \tag{4}$$

where A_p is the elastic work (mJ), P_E is the elastic potential (MPa, i.e. MJ/m^3), b is the width of the specimen (mm), h is the thickness of the specimen (mm) and l_o is the support span (mm).

To determine work in the plastic area we can use several procedures. It results from the work [6] that the plastic work for deflection can be determined by the integration of the regression equation (Eq.

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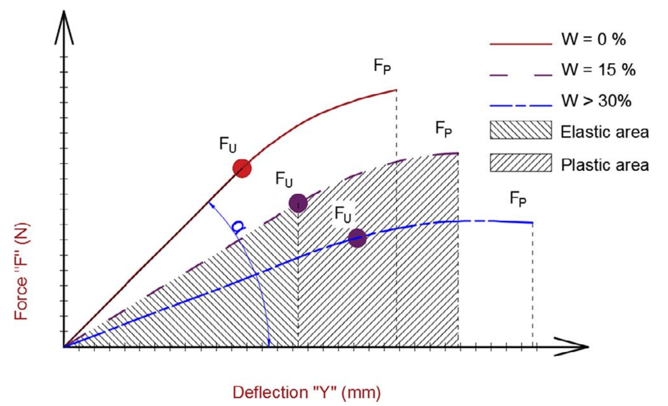

Fig. 1. Force – deflection diagram of wood in bending (where W is the moisture content).

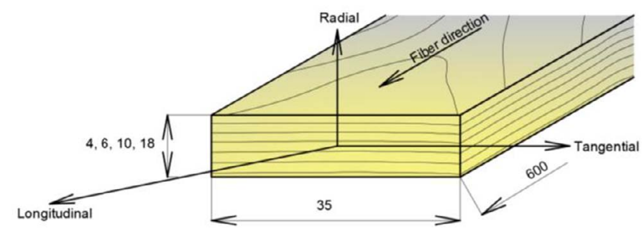

Fig. 3. The orthotropic direction of samples.

(5)) of the plastic area trend of the force-deflection diagram (Fig. 2A).

$$F(y) = a.\, y^2 + b.\, y + c \tag{5}$$

Integrating this equation within the limits y_E and y_P we obtain the equation

$$B_w^{-} = \frac{a}{3}.\, [y_P^3 - y_E^3] + \frac{b}{2}.\, [y_P^2 - y_E^2] + c.\, [y_P - y_E] \tag{6}$$

The given method of work is a little bit complicated as it requires the determination of coefficients a, b and c.

A simpler method is linear approximation (Fig. 2B) using the chored modulus between the proportionality limit "F_E" and the strength limit "F_P". The aproximated elastic work is calculated according to Eq. (7).

$$A_W = \frac{F_P + F_E}{2}(y_P - y_E) \tag{7}$$

where F_P is the force at the strength limit (N), F_E is the force at the proportionality limit (N), y_p is the deflection at the elastic limit (mm), and y_E the t the proportionality limit (mm).

Calculated values of approximated work (Fig. 2B) differ from the real values (Fig. 2A) by the so-called approximation error (Fig. 2C) which can be expressed in percentage according to Eq. (8).

$$\Delta W = \frac{B_W - A_W}{B_W}.\ 100\%. \tag{8}$$

In the past, the research dealt with the analysis of given material characteristics as well as with the influence of selected factors only marginally [7,8].

There is missing mainly the knowledge of the influence of the size wood specimen, degree of wood density or cyclic stress on the values of elastic work – total "B_W", approximated "A_W" or approximation error "ΔW" [9–11]. Therefore we consider necessary to pay attention to the given dependencies.

2. Material

Wood of European beech (*Fagus Sylvatica* L.) and common aspen (*Populus Tremula* L.) from the Poľana region in Slovakia was used for the preparation of the specimens. Lamellae of the dimensions of 4, 6, 10, and 18 mm thickness, 35 mm width, and 600 mm length were made of the selected species. The force was oriented parallel to grain (Fig. 3).

The specimens were conditioned to a moisture content of 8% in a climate chamber maintaining a relative humidity of 40% and temperature of 20 °C according to ISO 13061-1 [12]. The wood density was determined before and after testing according to ISO 13061-2 [13].

The cyclic loading was carried out on a cycler machine with cyclic bending of the test pieces using single-axis loading. The following numbers of cycles were selected for testing: 0 and 10,000. During the preliminary experimental testing, the test pieces were loaded with static bending to determine the breaking strength and proportionality limit because the test pieces had to be loaded up to 90% of the proportionality limit.

The acquired results were compared with those measured in test specimens subjected to 10% and 20% densification perpendicular to the grain. Ten test specimens were used for each test.

Table 1
Pressure applied in individual sets of test specimens.

Material Thickness (mm)	Degree of Densification 10%		Degree of Densification 20%	
	Beech (MPa)	Aspen (MPa)	Beech (MPa)	Aspen (MPa)
4	338	103	376	143
6	200	176	371	200
10	357	205	429	238
18	348	164	350	171

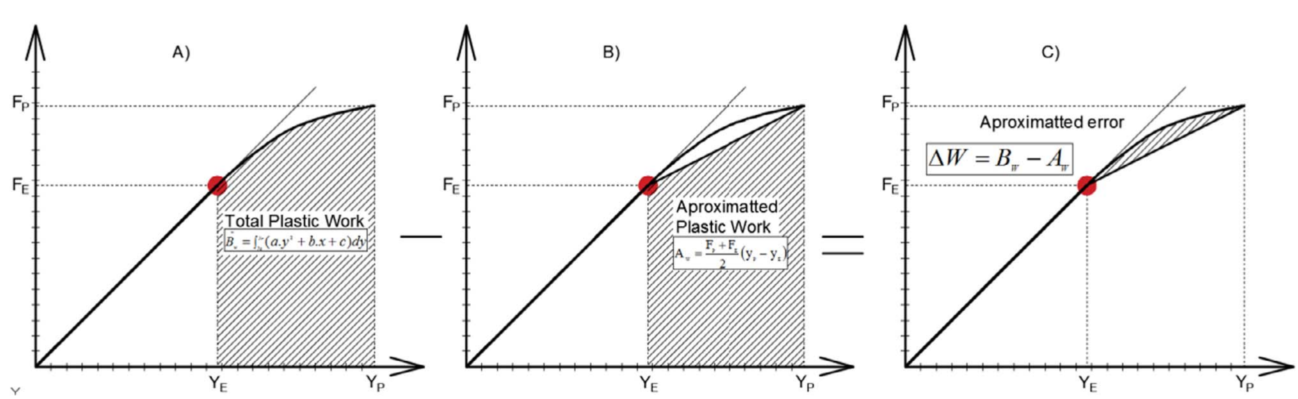

Fig. 2. Force – deflection diagram of wood in bending: A) Total plastic work "$\mathbf{B_W}$", B) Approximated plastic work "$\mathbf{A_W}$", C) Approximated error "**ΔW**".