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A novel framework using point interpolation method with voxels for variational asymptotic method unit cell homogenization of woven composites

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ABSTRACT

Micromechanical analysis of woven composites can be effectively carried out using variational asymptotic method (VAM) unit cell homogenization technique. The governing equations obtained by adopting this technique can be solved using numerical methods by conformal discretization of the domain. In the case of woven composites conformal discretization of the domain becomes difficult and time consuming. It is preferable to have a non-conformal discretization procedure for problems involving complex geometries like woven composites. A novel numerical framework for micromechanical analysis of woven composites based on VAM is proposed, where level-set method is used to define the interface as well as to decompose the domain into voxel regions of inclusions and matrix. Point interpolation method (PIM) is used to connect these voxel regions. The PIM-VOXEL framework thus developed is validated using examples having complex geometries taken from open literature for predicting elastic, thermal, thermo-elastic and visco-elastic properties. The proposed methodology alleviates the requirement for conformal meshing without compromising the accuracy and is capable of automation for homogenization and localization applications.

1. Introduction

The development of advanced materials play a crucial role in the advancement of technology. In the recent past a lot of scientific effort has been directed towards the development of new structural materials with desirable macro-level properties. These macro-level properties are a result of judicious mix of constituents, each having a different set of desirable properties at the micro-level. Here composite materials using textile preforms need special mention. These composites are generated by repeated/cyclic mechanical operation (weaving, braiding, stitching etc.); heterogeneous materials thus formed are periodic in nature.

Efficient design/simulation of structures made of heterogeneous materials requires characterization of these materials. The possibility of large set of heterogeneous material configurations that can be conceived make it prohibitive to carry out experimental characterization for each and every material. This challenge can be overcome by developing computational techniques based on the micromechanics of periodic heterogeneous materials for the determination of their effective properties. Such techniques should lend themselves for homogenization to predict macro-level properties and localization to determine local fields useful in material or structural failure analysis.

In general, homogenization of textile composites has been attempted using analytical [1] and numerical methods [2–5]. Numerical methods are predominantly based on finite element method (FEM), see [4,5]. These methods require refined conformal meshing of the unit cells to accurately predict the local stress and strain fields. For complex material architectures this becomes very time consuming. In [3] this is overcome by adopting a voxel based technique to automate the grid generation process. Voxel based technique has also been used to predict thermo-elastic properties in [4]. However, a series of finite element analyses (FEAs) were required to find the response of the cell to normal and shear strains and uniform temperature changes. Most of the methods mentioned above have been extended to predict coupled physical properties, however the models based on variational principles are noteworthy. Variational asymptotic method unit cell homogenization (VAMUCH) technique was used in [6–8] to predict the effective thermo-elastic, electromagnetoelastic and piezoelectric properties respectively. Here, the periodicity was considered as a small parameter and was used in expanding the energy functional asymptotically to obtain a variational statement for the unit cell. The minimization of this statement yielded the required governing equation and boundary conditions. The numerical implementation was done using FEM. VAMUCH

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has been successfully used to homogenize a wide class of heterogeneous materials. VAMUCH provides an efficient analytical tool to determine the properties by homogenization of a Repeating Unit Cell (RUC) of the composite material. When compared to FEM based methods, VAMUCH does not require loading of the RUC to determine the effective property and thus is highly efficient. FEM based implementation of VAMUCH requires conformal mesh where the element boundaries define the interface. Conformal meshing becomes difficult for complex geometries and hence there is a need for other numerical techniques to overcome this problem.

Automated meshing/analysis procedure can be developed based on non-conformal meshing techniques. In [9,10] VAMUCH based on XFEM framework is presented. The X-FEM/VAMUCH approach is applied successfully to many 2D examples reported in the VAMUCH literature. Numerical experiments on the homogenization of complex unit cells demonstrate the accuracy and simplicity of the XFEM/VAMUCH approach. The XFEM allows the use of meshes that are not necessarily matching the physical surface of the problem while retaining the accuracy of the classical finite element approach. For material interfaces, this is achieved by introducing Moës enrichment strategy[11]. In addition to the advantages of VAMUCH, XFEM reduces the meshing effort and allows for fully automated modeling of complex composite materials. A very thorough review of XFEM can be found in [12–14]; these articles present an overview of XFEM highlighting the methodological issues/solutions in its application. The enriched elements in XFEM techniques will have extra degrees of freedom (DOF) along with the regular DOF leading to large system matrix size. It becomes computationally expensive when used for 3D textile composites as the number of enriched elements grow with complex geometry. Unlike the standard XFEM, in [15] no additional unknowns are introduced at the nodes whose supports are crossed by discontinuities. An improved XFEM (iXFEM) has been developed in [16], where the extra DOF are not required for crack tip enrichment.

In most of the non-conformal methods, a level-set method is utilized to describe the geometry of the interfaces within a unit cell. This method of representing geometry with level-set is well established in XFEM, viz. [11] uses level-set to represent complex microstructure geometry. Similarly in [17], a level-set tool is developed to describe/generate geometry of textile reinforced composite. A methodology to model arbitrary holes and material interfaces using level-set has been proposed in [18]. The level-set method has also been used to represent the crack location, including the location of crack tips in [19]. In [20] a method for uncoupling geometrical description and approximation with the X-FEM was proposed. It is based on an uniform coarse mesh that defines a higher order approximation of the mechanical fields and an adapted mesh that defines the geometrical features by means of level-sets. In [21], a level-set method for the growth of non-planar three-dimensional cracks is presented.The crack is defined by two orthogonal level-sets whose intersection represents the crack front.

The authors in their previous work [22–25] have attempted to couple together the efficiency of VAMUCH and the advantage of voxelized geometry based discretization to develop a voxelized-VAM (VOXEL), which enables the usage of a highly efficient VAM on complex geometry, where conformal meshing is difficult. This framework proved to be highly efficient in the homogenization of complex geometry; however, it failed to give satisfactory result during the localization procedure.

In this paper, a level-set method is utilized to describe the geometry of the interfaces within a unit cell which is also used to obtain the fiber direction and normal at the interface of different phases. The voxelized-VAM is improved by incorporating meshfree point interpolation method (PIM)[26] in the voxels cut by the interface. Nodes are added at the interface determined by the level-set intersections with element edges and PIM is used to interpolate the field variable within the elements cut by interface. Even though most of the meshfree methods use approximants, PIM is a interpolant and has Dirac delta property that makes applying boundary conditions straight forward. The Dirac delta property of PIM makes the assembly of finite elements and PIM straight forward. This approach enables the non-conforming mesh to represent the inclusions in an appropriate manner and with reduced number of DOF when compared to XFEM.

The variational statement of the energy functional was solved following the methodology presented in [8] to determine the effective properties and to obtain the relation between local and global fields. Further, due to the variational structure of the problem the periodic boundary conditions were naturally obtained during the minimization process. Subsequently, PIM is used in conjunction with FEM in a non-conforming mesh to interpolate the fluctuating function.

Next section describes the level-set method, which is used to describe the geometry of fiber yarns inside the RUC, which is followed by a section on the decomposition of the RUC into PIM and FEM Zones, which forms the basic framework for the method proposed. Following the steps described in [27,6,28,29], the governing equations for the unit cell are derived in the next section. The proposed method is demonstrated by applying it on 2D as well as 3D problems available in literature [30] and the results are compared both for homogenization as well as the localization of the problems. Example problem to homogenize visco-elastic properties of woven composite is taken from [31]; for thermo-elastic properties, the experimentally tested woven composite from [32] is considered.

2. Description of the complex geometry using level-set function

The level-set function is defined as a signed distance function with respect to the yarn geometry, where the zero level-set represents the interface, negative level-set represents the inside of the yarn and the positive level-set represents the matrix phase. A typical level-set description of a reduced RUC for woven composite is shown in Fig. 1. In the present work, the complex fiber bundle architecture for each yarn is represented using level-set function. The elements which are crossed by zero level-set are chosen and decomposition of only that element into meshfree zone is done. The next section describes the procedure of decomposing the RUC into meshfree PIM zone and FEM zone.

3. Decomposition of RUC into PIM and FEM zone

A typical process of decomposition of RUC into PIM and FEM zone using level-set is depicted schematically in Fig. 2. In Fig. 2(a), ϕ is the interpolated nodal level-set value representing the inclusion, where $\phi = 0$ is the interface depicted as Γ. All the elements whose nodes have $\phi > 0$ or $\phi < 0$ are FEM zones – see Fig. 2(b). For all the elements which are crossed by $\phi = 0$, new nodes are added at the intersection of element edges with $\phi = 0$. The element crossed by $\phi = 0$ is removed and two PIM zones ($+$ and $-$) are formed. The shape function used for the field interpolation is derived from FEM in the FE zones, where as in the PIM zone it is derived from the meshfree PIM. The two set of nodes, one having positive level-set value and the other having negative level-set value are used to construct the meshfree shape function in the two PIM zones ($+$ and -). It may be noted that the new nodes which have zero level-set value are included in both zones.

Subsequent to the decomposition of RUC into PIM and FEM zones, the field variables are interpolated in the PIM zone using the PIM shape functions. The process of deriving the meshfree shape function using PIM is based on [26]. Consider a domain Ω (either $+$ or $-$) that is discretized using n nodes, for an arbitrary quadrature point x_Q. The function $u(x)$ is interpolated using the nodes in the domain as,

$$u(x,x_Q) = \sum_{i=1}^{n} p_i(x)a_i(x_Q) = \mathbf{p}^T(x)\mathbf{a}(x_Q) \tag{1}$$

where, $\mathbf{p}(x)$ is a polynomial basis. For a general 3D domain we have considered