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Effect of microstructure in thermoelasticity problems of functionally graded laminates

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A B S T R A C T

In this work the problem of thermoelasticity in composite, made of two different materials non-periodically distributed as microlaminas along one direction, is considered. The macroscopic properties of this composite are changing continuously along direction perpendicular to the laminas. In this note three models are presented: the tolerance and the asymptotic-tolerance model, taking into account the effect of the microstructure size and the asymptotic model, which equations omit this effect. To solve the equations of these three models, obtained by using the tolerance averaging technique, the finite difference method was used. Then the results obtained by the tolerance modelling were verified with the finite element method.

1. Introduction

The problem under consideration is a thermoelasticity issue in laminate, made of two different materials. These materials are non-periodically distributed as microlaminas along one direction, what is shown in Fig. 1. The macroscopic properties of this structure are changing continuously along direction x_1 – perpendicular to the laminas and this type of composites can be called the functionally graded laminates, cf. [1]. The thickness of the cells (*the microstructure parameter*) is constant and denoted by λ.

Thermoelastic phenomenon can be considered in relation to micromechanical models with idealized geometry, because the basic cell in reference to these laminates cannot be defined in a simple way. To analyse the various issues related to the functionally graded laminates, the assumptions of idealization similar to these used to analyse macroscopically homogeneous composites, can be applied. Between the methods, which are used for the periodic composites and can be adopted to describe the overall behaviour of the functionally graded laminates, the asymptotic homogenization and the homogenization based on the microlocal parameters, should be mentioned, cf. [2,3]. Moreover there are alternative methods, among other the higher order theory, cf. [4], which can be modified to describe this type of structures, but most of these methods do not take into account the effect of the microstructure size in analysed issues.

In order to obtain the averaged equations taking into account the effect of the microstructure size, the tolerance averaging technique was used, cf. [5], which give us the possibility to take into account this impact. This technique was used in many publications to consider various problems of thermoelasticity for composites with periodic structures, cf. [6–8] and for functionally graded laminates, cf. [9–11]. Furthermore the tolerance modelling was used among others to resolve thermal issues in a two-phase hollow cylinder, cf. [12,13], vibrations of periodic three-layered plates, cf. [14], dynamic problems for thin microstructured transversally graded shells, cf. [15], nonlinear vibrations of slender meso-periodic beams, cf. [16] and free vibration frequencies of thin functionally graded plates with one-directional microstructure, cf. [17]. In the analysis of various issues related to the composites and layered structures also the alternative methods can be used, for example the finite element method to analyse the elastic buckling of a sandwich beam, cf. [18], stability of three-layered annular plate, cf. [19], dynamic response control of layered plate, cf. [20] and stability of the sandwich band plate, cf. [21] or mathematical and numerical modelling for dynamic stability of sandwich beam, cf. [22], theoretical solutions to a problem of elastic three point-bending of a sandwich beam, cf. [23], strong form collocation method for solving laminated composite plates, cf. [24], generalized differential quadrature (GDQ) as numerical tool to analyse the laminated doubly-curved shells, cf. [25], layerwise theory and a differential quadrature finite element method in the analysis of composite plates, cf. [26] and modified couple stress theory to study free vibration of a Timoshenko functionally graded beam, cf. [27].

The basic aim of the application of the tolerance averaging technique is to replace the system of the differential equations with functional, highly-oscillating, tolerance-periodic and non-continuous coefficients, by equations with slowly-varying coefficients.

The main aim of this note is to obtain and present the equations of

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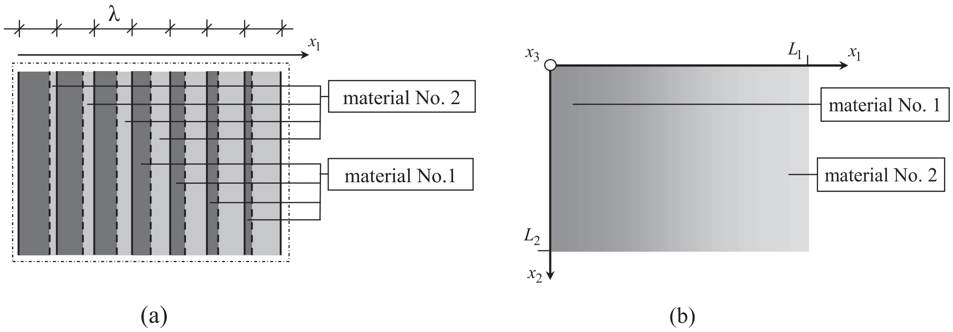

Fig. 1. The cross-section of considered laminate: (a) microstructure, (b) macrostructure.

three models for functionally graded laminates: the tolerance, the asymptotic-tolerance and the asymptotic model. The equations of these three models can be applied in the analysis of some specific cases, where the distribution of the ingredients is functional but non-periodic.

2. Modelling foundations

Thermoelastic phenomena for functionally graded laminates can be described by the known equations in the following form:

$$\partial_j(C_{ijkl}\partial_l u_k) - \rho\ddot{u}_i = \partial_j(b_{ij}\theta),$$
$$\partial_i(k_{ij}\partial_j\theta) - c\rho\dot{\theta} = T_0 b_{ij}\partial_j\dot{u}_i, \tag{1}$$

where $i, j, k, l = 1, 2, 3$, by u_i and θ the unknown displacements along the x_i-axis and the unknown temperature are denoted, respectively and the material coefficients (tensor of elasticity C_{ijkl}, tensor of heat conduction k_{ij}, tensor of thermal modules b_{ij}, mass density ρ, specific heat c) are non-continuous, highly-oscillating and tolerance-periodic.

The tolerance averaging technique is based on the many concepts which are *the averaging operation*, *tolerance-periodic*, *slowly-varying* and *highly-oscillating functions*, cf. [5]. The gradient of the function $f(x)$ is denoted by $\partial^i f$, where $x \in \Omega$, $i = 0, 1, 2$ and Ω is a space limited area in R. By $\Omega \times \Xi$ the space limited area in R^3 is denoted, where Ω is included in R and Ξ is included in R^2. The coordinates in Ω are denoted by $x = x_1$ or $z = z_1$ and coordinates in Ξ are denoted by $\varsigma = (\varsigma_1, \varsigma_2)$. The basic cell is defined as $\Delta \equiv [-\lambda/2, \lambda/2]$ and $\Delta(x) = x + \Delta$ is a cell with the centre in $x \in R$.

The averaging operator is defined by the following equation:

$$<\partial^i f>(x) \equiv |\Delta|^{-1}\int_{\Delta(x)} \widetilde{f}^{\,i}(x,z)dz, \tag{2}$$

where $i = 0, 1, 2$, $x \in \Omega$, $z \in \Delta(x)$ and a sign ~ is introduced to mark a periodic approximation of the gradient $\partial^i f$ in $\Delta(x)$.

The function f can be called *the tolerance-periodic function* in reference to the basic cell Δ and tolerance parameter δ, when the following terms are fulfilled:

$$(\forall x \in \Omega)(\exists \widetilde{f}^{(i)}(x,\cdot) \in H^0(\Delta))(\|\partial^i f|_{\Omega_x}(\cdot) - \widetilde{f}^{(i)}(x,\cdot)\|_{H^0(\Omega_x)} \leqslant \delta), \int_{\Delta(\cdot)} \widetilde{f}^{(i)}(\cdot,z)dz \in C^0(\overline{\Omega}), \tag{3}$$

where i accepts values 0, 1, 2 and $H^0(\Delta)$ is a space of Δ-periodic functions, which can be square integrable.

The function u can be called *the slowly-varying function* in reference to the basic cell Δ and tolerance parameter δ, when the function u is a *tolerance-periodic function* and the succeeding term is fulfilled:

$$(\forall x \in \Omega)(\widetilde{u}^{(i)}(x,\cdot)|_{\Delta(x)} = \partial^i u(x)), \tag{4}$$

where $i = 0, 1, 2$ and a periodic approximation of $\partial^i u(\cdot)$ in an area of $\Delta(x)$ for every $x \in \Omega$ is a constant function.

The function h can be called *the highly-oscillating function* in reference to the basic cell Δ and tolerance parameter δ, if this function is a *tolerance periodic function* and the following term is fulfilled:

$$(\forall x \in \Omega)(\widetilde{h}^{(i)}(x,\cdot)|_{\Delta(x)} = \partial^i \widetilde{h}(x)), \tag{5}$$

where $i = 0, 1, 2$.

3. Modelling procedures

The tolerance averaging technique is based on some assumptions. The main assumption of this technique is *the micro-macro decomposition*, where the basic unknowns can be taken as sums of the averaged parts and the oscillating parts, according to the following equations:

$$u_i(x,\varsigma,t) = w_i(x,\varsigma,t) + h^A(x)v_{iA}(x,\varsigma,t),$$
$$\theta(x,\varsigma,t) = \vartheta(x,\varsigma,t) + g^B(x)\psi_B(x,\varsigma,t). \tag{6}$$

On the other hand the oscillating part can be expressed as a product of a known fluctuation shape functions and the fluctuation amplitudes, which are the new basic unknowns. The basic unknowns in this case are the displacements u_i (i accepts values 1, 2, 3) and the temperature θ, the new basic unknowns are the fluctuation amplitudes of the displacements v_{iA} and the temperature ψ_B. Both the displacements, the temperature and the fluctuation amplitudes are *the slowly-varying functions* of the coordinate x_1. By h^A and g^B the known fluctuation shape functions of the displacements and the temperature, respectively, are denoted. The fluctuation shape functions have to be defined for each analysed case. Basing on the available literature, cf. [28], concerning on the thermal issues in functionally graded laminates, one fluctuation shape function is assumed ($A = 1$, $B = 1$), expressed by the following equation:

$$h(x,z) = g(x,z) = \begin{cases} -\lambda\sqrt{3}/v_1(x)(2z/\lambda + v_2(x)) & \text{for} \quad z \in (-\lambda/2, -\lambda/2 + v_1(x)\lambda) \\ \lambda\sqrt{3}/v_2(x)(2z/\lambda - v_1(x)) & \text{for} \quad z \in (-\lambda/2 + v_1(x)\lambda, \lambda/2) \end{cases}, \tag{7}$$

where $v_1(x)$ and $v_2(x)$ define the share of the first and the second material in the cell and describe the gradation of properties of the laminate. This type of the fluctuation shape function guarantees the continuity of the displacements and the temperature between the layers and between the sublayers.

The second assumption of the tolerance averaging technique is the periodic approximation of k^{th} derivatives of functions of the displacements and the temperature, which can be defined using the following equations:

$$\widetilde{u}_i^{(k)}(x,z,\varsigma) = \nabla^k w_i(x,\varsigma) + \partial^k \widetilde{h}^A(x,z)v_{iA}(x,\varsigma) + \widetilde{h}^A(x,z)\overline{\nabla}^k v_{iA}(x,\varsigma),$$
$$\widetilde{\theta}^{(k)}(x,z,\varsigma) = \nabla^k \vartheta(x,\varsigma) + \partial^k \widetilde{g}^B(x,z)\psi_B(x,\varsigma) + \widetilde{g}^B(x,z)\overline{\nabla}^k \psi_B(x,\varsigma). \tag{8}$$

where $z \in \Delta(x)$, $x \in \Omega$.

To obtain the equations of the tolerance model, the orthogonalization method is used. Following this method, the approximated functions of the displacements are expanding in series relative to the linear-