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ABSTRACT

The influence of microstructure on macro-structural plate bending is incorporated into plate bending theories by replacing the classical elasticity theory by strain gradient elasticity. The microstructure is represented by gradients of macro-strains and one new material coefficient which is called the micro-scale length parameter. The strain gradients can be introduced either in the non-symmetrized or in the symmetrized formulation. Though the former is physically inconsistent, both these formulations are considered and compared in numerical simulations. The governing equations as well as boundary conditions are derived from the principle of variations applied to unified formulation of plate bending with assuming the assumptions of the classical thin plate bending theory (Kirchhoff-Love theory) and/or the shear deformation plate theory of the 1st and 3rd order (FSDPT and TSDPT). The derived formulation is applicable to FGM (Functionally Graded Material) plates, since the gradation of Young's modulus is allowed in both the transversal and in-plane directions. Some numerical examples are considered in numerical simulations, in order to illustrate the influence of microstructure on the behavior of macro-structural plate subjected to static transversal loading.

Key words: Thin and/or Thick plate; Strain gradients; Micro-scale length parameter; Functionally graded materials; Size effect

1. Introduction

Small-scale structural elements such as beams, plates and shells are often used as components in micro- and nano-electromechanical systems (MEMS and NEMS). Classical continuum theories (including elasticity) have been initially developed to describe physical phenomena and processes that could be captured by naked eye, i.e. ranging roughly from millimeters to meters. Nevertheless, there are known applications of continuum theories to earth scale (faults and earthquakes) and astronomic scales (relativistic elastic solids) as well as to phenomena evolving at atomistic scale (e.g. elastic theory of dislocations; characterization of deformation behavior of nanotubes, nanowires, grapheme sheets, etc. at the nanoscale) [1]. However, recent experimental observations with newly developed nano-indenters and atomic force microscopes revealed that classical continuum theories do not suffice for an accurate and detailed description of deformation phenomena in the regime of microns and nanoscales. Among the problems which cannot be treated in classical elasticity, one can name, for instance, the classical elastic singularities occurring at point loads, at dislocation lines and crack tips, the size effects which become dominant as the specimen size decreases, etc. The inability of standard continuum mechanics theories to deal with above problems is due to the absence of an internal length parameter (characteristic of microstructure) in constitutive equations, because the material coefficients in classical elasticity are taken from experiments for macrostructures. To overcome this insufficiency of the classical elasticity, a great effort has been done mainly in two directions: (i)| atomistic or molecular simulations; (ii) generalized continuum (phenomenological) theories. The molecular dynamic (MD) simulations are too computational expensive and inappropriate for study the behavior of macrostructures. The development of generalized continuum theories in 1960' was stimulated by 50 years old work of brothers Cosserat [2] on micropolar continuum. A thorough survey of the notational systems used in micropolar elasticity including the experimental efforts taken to measure micropolar elastic moduli has been presented in [30]. Recently, a comprehensive review of the works on continuum mechanics models for capturing size effects in small-scale structures has been given by Huu-Tai Thai et al [3], Askes and Aifantis [1]. Tekoglu and Onck [31] presented a transparent classification of generalized continuum theories by splitting them into the higher-grade theories (in which the degrees of freedom are given by displacements and deformation measure is given by strains and higher order derivatives of displacements) and the higher-order theories (some additional degrees of freedom are employed besides displacements). The further subdivision of each class of generalized continuum theories is given according to utilization of deformation measures. Tekoglu and

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