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Fundamental frequency of fully clamped antisymmetric angle-ply laminated plates with structural anisotropy

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ABSTRACT

An analytical solution determining the fundamental frequency of a fully clamped composite anisotropic laminated plate is presented in the paper. The plate is composed of unidirectional composite plies oriented at some angle to one of the plate sides. The plies alternating over the plate thickness differ from each other by only the sign of the angle of orientation. Such a plate is characterised by the structural anisotropy with the extension-twisting and bending-shear coupling effects which are taken into account in the appropriate constitutive equations. The governing equations model the coupled in-plane and out-of-plane plate motions. The vibration problem is solved using the Galerkin method. The beam functions corresponding to the first vibration mode of a beam with clamped ends are employed as the approximating functions. The problem is reduced to a solution of cubic algebraic equation. Based on this solution, effects of the angle of reinforcement orientation and number of plies on the fundamental frequency of the plate with structural anisotropy are investigated. The results are verified using finite element method. An assessment of the anisotropy effect on the frequency value has been performed by comparison with the results obtained based on the orthotropic model of the plate. The formula providing the number of plies for a plate with structural anisotropy is derived for the prescribed fundamental frequency.

1. Introduction

One of the typical properties of composite materials is that they are created concurrently with the structure. Since the mechanical characteristics of these materials, determined by their reinforcement arrangements, can vary in rather wide ranges, the composite structures can be designed with the properties meeting operational requirements and loadings. Composite laminated rectangular plates which are widely used as load bearing structural components in various industrial applications are typical examples of such structures. There is a large variety of laminated structures of these plates depending on their structural role and loading conditions.

Often, the laminated plates are composed of unidirectional composite plies. Modern automated manufacturing processes are capable of laying these plies sequentially at the prescribed angle of orientation to one of the plate sides [1]. There are laminated plates in which the adjacent plies differ from each other only by the sign of the angle of the fibre orientation. Such plates are characterised by structural anisotropy with the extension-twisting and bending-shear coupling effects. These effects are discussed in the monographs published by Jones [2], Vinson and Sierakowski [3], Vasiliev and Morozov [4], and Kollar and Springer [5] and exhibit themselves in the coupled in-plane and out-of-plane plate deformations. Mechanical behaviour of the plates demonstrating such effects has been analysed by Dhurvey and Mittal [6], Tsai [7], Herencia et al. [8], York and Weaver [9], Li and York, [10,11], and York [12–15].

Growing applications of composite laminated plates made of unidirectional plies in various non-conventional structural designs call for further studies of their behaviour and structural response determined by their structural anisotropy. In such designs, there is a lot of practical interest in the calculation of the fundamental frequency of the fully clamped anisotropic plates under consideration. This is because the values of the fundamental frequency are often used as stiffness criteria for these plates in the design procedures. It is always advantageous to have an analytical formula for the rapid assessment of the fundamental frequency of a structure that could be used in the design process and would not be computationally expensive.

Free vibration analyses of anti-symmetric angle-ply laminated plates have been reported in many publications. For example, Gorman and Ding carried out numerical free vibration analysis of clamped antisymmetric angle-ply

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laminated rectangular plates using Superposition-Galerkin method [16]. They introduced a superposition of the building block solutions for the plates with combinations of clamped and simply supported edges to numerically solve the problem. Net contribution of the building blocks towards edge rotation along each of the four edges of the plate were expanded in an appropriate series. Standard numerical procedures were utilised to solve resulting eigenvalue problems. Qatu [17] employed the Ritz method with algebraic polynomial displacement functions to solve the vibration problem for laminated composite plates having different boundary conditions. Natural frequencies and mode shapes for plates having two adjacent free edges and the remaining edges either simply supported, clamped or free have been considered. Aygodu and Timarci [18] performed vibration analyses of cross-ply laminated square plate with general boundary conditions. The free vibration frequencies were obtained by applying the Ritz method where the five displacement components were presented as the series of algebraic polynomials. The resulting matrix equation with the number of equations depending on the number of terms employed in all of the series representing the generalised eigenvalue problem was solved numerically. Shi et al. [19] expanded the unknown displacements into double Fourier series and applied the Galerkin method. This resulted in a set of $5 \times M \times N$ (where M and N are the numbers of the retained terms in the Fourier series) linear algebraic equations in the form of an eigenvalue problem from which approximate values of frequency parameters were determined using standard numerical procedures. Viswanathan et al. [20] studied free vibrations of anti-symmetric angle-ply laminated rectangular plates with two opposite sides simply supported and another two clamped. They used Radial Basis function method and spline approximation to reduce the problem to the generalised eigenvalue problem that was solved numerically. A generalised eigenvalue problem was obtained for the similar plates with variable thickness and solved numerically for an eigenfrequency parameter and an associated eigenvector of cubic spline coefficients by Javed et al. [21]. In all these studies, the approximate solutions were obtained using numerical methods applied to large systems of equations.

In this work, for the first time an analytical solution determining the fundamental frequency of a fully clamped composite antisymmetric angle-ply anisotropic laminated plate and not requiring an application of numerical methods has been obtained. The plate is composed of unidirectional composite plies oriented at some angle to one of the plate sides. The alternating over the plate thickness plies differ from each other by the sign of the angle of orientation. In the analysis, the structural anisotropy of the plate characterised by the extension-twisting and bending-shear coupling effects is taken into account in the constitutive equations. The plate vibrations are modelled by the governing equations reflecting the coupled in-plane and out-of-plane plate motions. This system of equations is of eight order with respect to each of coordinates and is solved using the Galerkin method. The beam functions corresponding to the first vibration mode of a beam with clamped ends are employed as the approximating functions. As a result, the problem is reduced to a solution of cubic algebraic equation written in terms of unknown frequency. The least root of this equation found by Cardano's method provides the value of the plate fundamental frequency. Based on this solution, effects of the angle of reinforcement orientation and number of plies on the fundamental frequency of the plate with structural anisotropy have been investigated. The results are verified using finite element method. An assessment of the anisotropy effect on the frequency value has been performed by comparison with the results obtained based on the orthotropic model of the plate. The formula determining the number of plies in a plate with structural anisotropy for the desired value of the fundamental frequency is derived using the aforementioned solution obtained in this work.

2. Constitutive equations

Consider a composite laminated rectangular plate composed of $2k$ plies. All the plies have the same thickness δ. The middle plane of such a plate having the total thickness of $h = 2\delta k$ is positioned at the distance $h/2 = \delta k$ from the top and bottom surfaces. Refer the middle plane to a Cartesian coordinate frame xyz (see Fig. 1). The plate dimensions along the axes x and y are a and b, respectively. The plate is composed of unidirectional plies of composite material which are placed at angles $+\phi$ and $-\phi$ to the axis x as shown in Fig. 2. The composite material can be treated as orthotropic in the principal material axes related to the reinforcement direction. The mechanical characteristics of the ply are determined by the moduli of elasticity along and across the fibre directions E_1 and E_2, in-plane shear modulus G_{12} and Poisson's ratios ν_{12} and ν_{21}. The latter satisfy the symmetry condition: $E_1\nu_{12} = E_2\nu_{21}$. However, as known, the ply material in the coordinate frame xy is anisotropic [4]. For such a ply, the elastic properties are defined by nine parameters A_{mn} (m, n = 1, 2, 3), which are calculated in terms of the elastic properties of unidirectional material and angle ϕ as follows

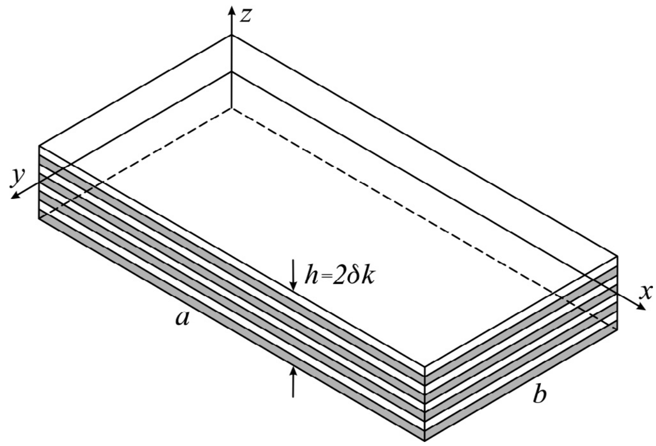

Fig. 1. Laminated plate.

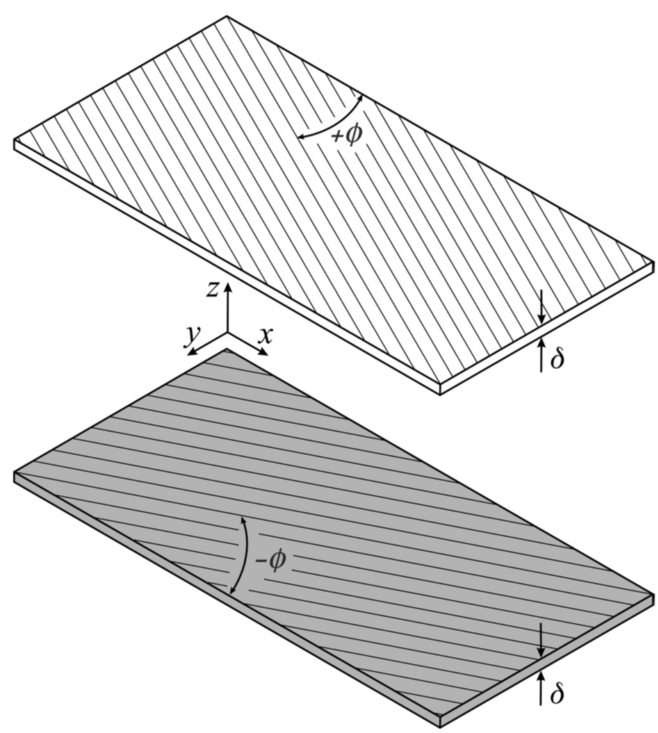

Fig. 2. Adjacent layers.

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