ARTICLE IN PRESS

Composite Structures xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Unsymmetrical sandwich beams under three-point bending – Analytical studies

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ABSTRACT

The main purpose of the work is an analytical description of a simply supported three-layer beam with facings of various thicknesses and various material constants. The beam core is made of metal foam. Its mechanical properties are described. The beam is subjected to three-point bending. The analytical model of the beam is formulated based on a nonlinear hypothesis of deformation of the cross section of the beam. The proposed hypothesis is a generalization of the classical one – the broken line hypothesis. The linear relationship between the strains and displacements is assumed. Stresses are defined according to the Hooke's law. Furthermore, the elastic strain energy and the work of the load is defined. Based on the principle of the total potential energy the system of equilibrium equations is derived. Afterwards, the system is analytically solved with the use of trigonometric series. The maximum deflections and shear stresses are obtained. Moreover, the position of the neutral axis is determined. The calculations for the family of beams are carried out. The results are compared to FEM (Finite Element Method) solutions obtained from SolidWorks Simulation system.

1. Introduction

The classical sandwich structures are symmetrical. Vinson [1] presented a review of the papers of the 20th century, related to the sandwich structures. Carrera [2] described a review of multilayer structure modelling with consideration of the zig-zag theories. Chakrabarti et al. [3] presented analysis of laminated sandwich beam with soft core, based on higher order zig-zag theory. Magnucka-Blandzi [4] studied stability and static problems of a sandwich beam with a metal foam core, using three hypotheses. Magnucka-Blandzi [5] presented the mathematical modelling of a rectangular sandwich plate with a metal foam core. Jiang et al. [6] presented the problem of large deflection of a sandwich beam under three-point bending, with consideration of the failure mechanism. Chen et al. [7] analysed free vibration of shear deformable sandwich beam with a porous core of varying mechanical properties. Caliri Jr et al. [8] presented a review of plate and shell theories applied to laminated and sandwich structures, with special attention paid to the Finite Element Method. Moreno et al. [9] described behaviour of unidirectional carbon fibre in a three-point bending test. Magnucki et al. [10] theoretically studied bending and buckling problems of a steel composite beam with corrugated main core and sandwich faces. Magnucka-Blandzi and Rodak [11] presented comparative analysis of bending and buckling of a metal seven-layer beam with lengthwise corrugated main core with classical sandwich beam. Morada et al. [12] described the failure mechanism of a sandwich beam with an ATH/epoxy core under static and dynamic three-point bending. Sayyad and Ghugal [13] presented an extensive critical review of the papers devoted to bending, buckling and vibration of laminated and sandwich beams. Magnucka-Blandzi [14] focused on comparative analysis of bending and buckling of a metal seven-layer beam with crosswise corrugated main core with classical sandwich beam. Smyczynski and Magnucka-Blandzi [15] compared the effect of two adopted nonlinear hypotheses on the results obtained for three-point bending of a sandwich beam, with two binding layers. Abrate and di Scoiva [16] presented a review of equivalent single layer theories for composite and sandwich structures. Magnucki et al. [17] analysed the three-point bending of a short beam with symmetrically varying mechanical properties.

The subject of the study is a simply supported unsymmetrical sandwich beam of length L and width b. The beam is under three-point bending. Faces of the beam are of different thicknesses t_{f1}, t_{f2} and are made of different materials with Young's modules E_{f1}, E_{f2}. The core of thickness t_c is made of material with elastic modules E_c, G_c. (Fig. 1).

The x axis is co-linear with the neutral axis. Due to unsymmetrical beam structure the neutral axis is shifted by y_0 with regard to geometrical symmetry axis of the core. The novelty of the presented research

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https://doi.org/10.1016/j.compstruct.2018.02.086
Received 20 December 2017; Received in revised form 19 February 2018; Accepted 27 February 2018

Please cite this article as: Magnucka-Blandzi, E., Composite Structures (2018), https://doi.org/10.1016/j.compstruct.2018.02.086

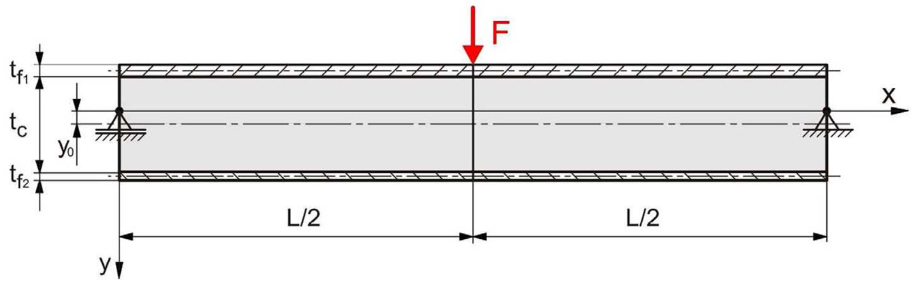

Fig. 1. Scheme of the unsymmetrical sandwich beam and load.

consists in analytical description of an unsymmetrical sandwich beam with consideration of the elastic strain energy. In particular, the position of the neutral axis is determined, which is not identical to geometric middle axis of the beam. It may be noticed that sandwich beams are recognized as composite structures. The proposed nonlinear hypothesis (power functions) is assumed for modelling of the beam. Various hypotheses and theories were used to study the bending of composite beams by other authors. For example, a modified couple stress theory and a meshless method were used by Roque et al. [18], a layer-wise third order shear and normal deformable plate/shell theory by Batra and Xiao [19], a first-order shear deformation theory by Carpentieri et al. [20], the hypotheses of the Grigolyuk-Chulkov and the modified couple stress theory by Awrejcewicz et al. [21].

2. Analytical model of the beam

The nonlinear hypothesis is assumed for modelling of the beam. In result of deformation of the plane cross section the straight line before bending transforms into a curve-line (Fig. 2).

Displacements in the subsequent layers of the beam are as follows

- the upper face

$$-\left(x_{f1}+\tfrac{1}{2}-\eta_0\right)\leqslant\eta\leqslant-\left(\tfrac{1}{2}-\eta_0\right)$$
$$u(x,y)=-t_c\left[\eta\tfrac{dv}{dx}+\left(\tfrac{1}{2}-\eta_0\right)\psi_0(x)\right],\tag{1}$$

- the core

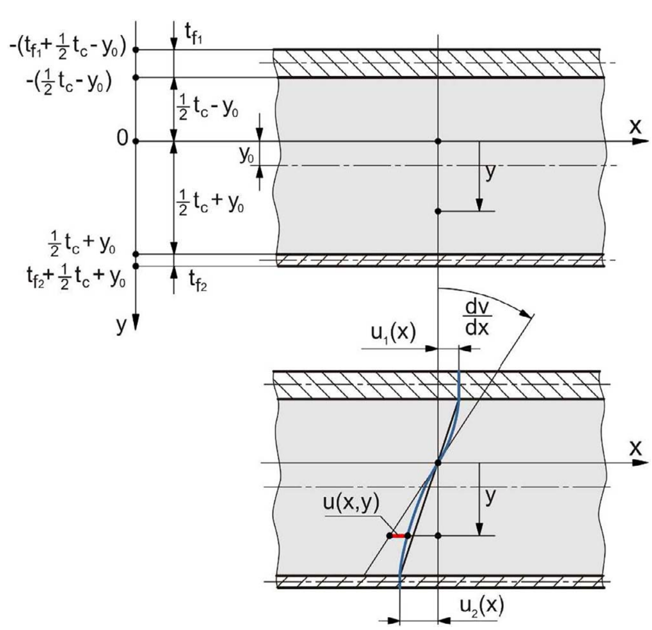

Fig. 2. The deformation of plane cross section of the beam – the nonlinear hypothesis.

$$-\left(\tfrac{1}{2}-\eta_0\right)\leqslant\eta\leqslant\tfrac{1}{2}+\eta_0$$
$$u(x,y)=-t_c\left\{\eta\left[\tfrac{dv}{dx}-\psi_0(x)\right]+\eta\left[(\eta-\eta_0)^2-\tfrac{1}{4}\right]\psi_1(x)\right\},\tag{2}$$

- the lower face

$$\tfrac{1}{2}+\eta_0\leqslant\eta\leqslant x_{f2}+\tfrac{1}{2}+\eta_0$$
$$u(x,y)=-t_c\left[\eta\tfrac{dv}{dx}-\left(\tfrac{1}{2}+\eta_0\right)\psi_0(x)\right],\tag{3}$$

where: $v(x)$ – deflection, $\eta=y/t_c$ – dimensionless coordinate, $\eta_0=y_0/t_c$, $x_{f1}=t_{f1}/t_c$, $x_{f2}=t_{f2}/t_c$ – dimensionless parameters, and $\psi_0(x)$, $\psi_1(x)$ – dimensionless functions of the shear effect.

Taking into account the linear dependence for the function $\psi_0(x)$, the displacements in the upper and lower faces are

$$u_1(x)=t_c\left(\frac{1}{2}-\eta_0\right)\psi_0(x),\quad u_2(x)=t_c\left(\frac{1}{2}+\eta_0\right)\psi_0(x).\tag{4}$$

Therefore, longitudinal and shear strains in the layers of the beam are as follows:

- the upper face

$$\varepsilon_x^{(u-f)}=\frac{du}{dx}=-t_c\left[\eta\frac{d^2v}{dx^2}+\left(\frac{1}{2}-\eta_0\right)\frac{d\psi_0}{dx}\right],\quad\gamma_{xy}^{(u-f)}=\frac{du}{dy}+\frac{dv}{dx}=0,\tag{5}$$

- the core

$$\varepsilon_x^{(c)}=\tfrac{du}{dx}=-t_c\left\{\eta\left(\tfrac{d^2v}{dx^2}-\tfrac{d\psi_0}{dx}\right)+\eta\left[(\eta-\eta_0)^2-\tfrac{1}{4}\right]\tfrac{d\psi_1}{dx}\right\},$$
$$\gamma_{xy}^{(c)}=\tfrac{du}{dy}+\tfrac{dv}{dx}=\psi_0(x)-\left(3\eta^2-4\eta_0\eta+\eta_0^2-\tfrac{1}{4}\right)\psi_1(x),\tag{6}$$

- the lower face

$$\varepsilon_x^{(l-f)}=\frac{du}{dx}=-t_c\left[\eta\frac{d^2v}{dx^2}-\left(\frac{1}{2}+\eta_0\right)\frac{d\psi_0}{dx}\right],\quad\gamma_{xy}^{(l-f)}=\frac{du}{dy}+\frac{dv}{dx}=0,\tag{7}$$

The stresses (Hooke's law) for these layers:

$$\sigma_x^{(u-f)}=E_{f1}\varepsilon_x^{(u-f)},\quad\tau_{xy}^{(u-f)}=0,\tag{8}$$

$$\sigma_x^{(c)}=E_c\varepsilon_x^{(c)},\quad\tau_{xy}^{(c)}=G_c\gamma_{xy}^{(c)},\tag{9}$$

$$\sigma_x^{(l-f)}=E_{f1}\varepsilon_x^{(u-f)},\quad\tau_{xy}^{(l-f)}=0.\tag{10}$$

The elastic strain energy of the beam

$$U_\varepsilon=U_\varepsilon^{(u-f)}+U_\varepsilon^{(c)}+U_\varepsilon^{(l-f)},\tag{11}$$

where:

- the elastic strain energy of the upper face after integration with regard to its thickness