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Uncertain material properties on wave dispersion behaviors of smart magneto-electro-elastic nanobeams

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ABSTRACT

Uncertainties in material properties caused by the small-scale length effect are serious for nanostructures, which may affect their mechanical responses accordingly. This paper devotes to studying the effect of uncertain material properties on wave propagation characteristics of magneto-electro-elastic nanobeams subjected to external electric and magnetic fields. Based on the nonlocal Euler-Bernoulli beam theory, the governing differential equations of motion are derived by using the Hamilton's principle. Considering limited experimental data, uncertain-but-bounded parameters are employed to quantify the uncertain material properties including elastic constants, mass density, piezoelectric, piezomagnetic, dielectric, magnetoelectric and magnetic constants. A high precision interval analysis method is presented to evaluate the upper and lower bounds of the wave dispersion curves. Meanwhile, the presented method is validated with Monte-Carlo simulation, and its validation is also demonstrated by comparing with probabilistic method. Numerical results suggest the effect of uncertainties in material properties is significant in understanding the wave dispersion behaviors of magneto-electro-elastic nanobeams. The presented method can serve as an effective tool to quantify the dynamic response of nanosensors and nanoactuators with uncertain material properties.

1. Introduction

Nanostructures have attracted a great deal of attention due to their unique mechanical, electrical, thermal, and structural properties, which enable them to be successfully used in many applications including micro-electro-mechanical sensors, transistors, actuators, probes and resonators [1]. Therefore, it is crucial to investigate the mechanical and physical behaviors of nanostructures. Many studies have shown that the small scale effect becomes significant in nanostructures and consequently the classical theory cannot predict this size dependent behavior efficiently [2]. There are two mainly methods i.e., atomistic and non-classical continuum mechanic approaches, to study the small scale effect of nanostructures. Because the atomistic approach is computationally expensive and time consuming for large-scaled structures, the non-classical continuum mechanic approaches are always the first choice in many cases [3,4].

Several non-classical continuum mechanic approaches including the nonlocal elasticity [5–14], strain gradient [15,16], modified strain gradient [17–21], couple stress [22] and modified couple stress [23–25] theories have been developed to describe the size effect of nanostructures. The nonlocal elasticity theory is the most commonly used approach, which was initially proposed by Eringen [5] and was subsequently introduced to nano-materials by Peddieson et al. [26]. In this theory, the small scale effect is captured by assuming the stress at any location to be a function of the strain field at every point of the whole body. Recently, the smart nanostructures are considered as the promising candidates for the future nano-electro-mechanical systems (NEMS) due to their excellent magneto-electro-elastic performances. Especially, the magneto-electro-elastic nanobeam is widely used as an elementary component in NEMS. Various nonlocal beam theories i.e., nonlocal Euler-Bernoulli beam, nonlocal Timoshenko beam and nonlocal higher-order shear deformation theories are devoted to understanding the wave propagation, buckling and vibration behaviors of such structures. For example, Arefi and Zenkour [27] applied the nonlocal Timoshenko beam model for analyzing the wave propagation properties of functionally graded magneto-electro-elastic nanobeams. Ke et al. [28,29] investigated the wave propagation and vibration behaviors of magneto-electro-elastic nanobeams based on the nonlocal Euler-Bernoulli and Timoshenko beam theories. Ebrahimi et al. [30–33] used the nonlocal higher-order shear deformation theory to study the static and dynamic responses of magneto-electro-elastic nanostructures. Other related studies on magneto-electro-elastic nanostructures also can be found in Refs. [34–36].

It also should be noted that most aforementioned studies are

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conducted under the assumption of deterministic material properties. Due to the nano-scale effect of nanostructures, many factors like manufacturing tolerances, physical imperfections, measurement errors, model inaccuracies and system complexities are unavoidable and will lead to serious uncertainties [37]. The material uncertainty at nanoscale was reported by many researchers, for instance, Salvetat et al. [38] measured the Young's modulus and shear modulus of single-walled carbon nanotubes (CNTs) by using the atomic force microscope (AFM) and 50% of error was found in their measurement results. Wong et al. [39] also adopted the AFM to measure the Young's modulus of CNTs and their results were distributed in the range of 1.28 ± 0.59 TPa. Employing the transmission electron microscope, Treacy et al. [40] found the Young's modulus of CNTs was in a wide range from 0.40 TPa to 4.15 TPa, while ± 30% error was presented in the measurement results of Enomoto et al. [41]. Following their experimental results, one can find that the material properties of nanostructure should be described as uncertain parameters.

Probabilistic analysis method (PAM) is one of the most popular approaches for the uncertain problems of nanostructures, where the probability density function (PDF) should be defined unambiguously [42–44]. By using this method, Chang [45,46] suggested a stochastic finite element method for nonlinear vibration problem of fluid-loaded double-walled CNTs with random material parameters. Scarpa and Adhikari [47] presented a stochastic reduced order model for solving the natural frequencies of a single-walled CNT with random flexural modulus, thickness and mass density. In their paper, because there was no sufficient experimental data in the open literatures to evaluate the accurate PDF of flexural modulus, an equivalent atomistic finite element model was used to make that come true. To overcome the massive experimental data needed in probabilistic model, the interval analysis method (IAM) has aroused widely concern. This method only requires the well-defined bounds of the uncertain parameters rather than enough information about their PDF. The IAM is widely used in the structural problems at macro-scales [48–52], while its application in nanostructures is limited. In recent studies, Chen et al. [53] adopted an interval homogenization-based method to evaluate the lower and upper bounds of elastic properties of periodic microstructure. Radebe and Adali [54,55] investigated the buckling problem of nanoplates with uncertain-but-bounded material parameters.

The investigation on uncertain wave propagation is meaningful to understand the dynamic behaviors of structures and it can provide helpful guidelines for structural ultrasonic evaluation and health monitoring. Hosseini and Shahabian [56] studied the wave propagation in functionally graded materials with random uncertain constitutive mechanical properties. Nguyen et al. [57] presented a probabilistic framework for investigating ultrasonic wave reflection and transmission through an anisotropic elastic plate with uncertain material properties. Liu et al. [58,59] studied the effects of uncertain material properties on the transverse and longitudinal wave propagation in CNTs. However, no detailed investigation has been reported in wave propagation analysis of magneto-electro-elastic nanostructures with material uncertainties.

This paper aims to set up a theoretical model to analyze the wave dispersion characteristics of magneto-electro-elastic nanobeams with uncertain material properties. The material parameters including elastic constants, mass density, piezoelectric, piezomagnetic, dielectric, magnetoelectric and magnetic constants are all regarded as uncertain-but-bounded parameters. The upper and lower bounds of the wave dispersion curves are investigated under different uncertain levels by using the IAM. Also, the combined influences of the material uncertainties, the small scale coefficient, as well as the external electric and magnetic fields on the wave frequency and phase velocity are discussed. Furthermore, the PAM and Monte Carlo simulation (MCS) are presented to validate the proposed IAM.

2. Equation of wave propagation

2.1. Nonlocal elasticity

Because the structural dimension of the magneto-electro-elastic nanobeam is comparable to its internal length scale, the size-dependent effect is significant. Eringen's nonlocal elasticity theory is widely adopted to describe the size-dependent effect of nanobeams [5]. According to this theory, the basic equations for a nonlocal magneto-electro-elastic solid can be expressed as

$$\sigma_{ij} = \int \alpha(|x-x'|,\tau)[c_{ijkl}\varepsilon_{kl}(x') - e_{mij}E_m(x') - q_{nij}H_n(x')]\,\mathrm{d}V(x') \tag{1a}$$

$$D_i = \int \alpha(|x-x'|,\tau)[e_{ikl}\varepsilon_{kl}(x') + s_{im}E_m(x') + d_{in}H_n(x')]\,\mathrm{d}V(x') \tag{1b}$$

$$B_i = \int \alpha(|x-x'|,\tau)[q_{ikl}\varepsilon_{kl}(x') + d_{im}E_m(x') + \chi_{in}H_n(x')]\,\mathrm{d}V(x') \tag{1c}$$

$$\sigma_{ij,j} = \rho\ddot{u}_i,\ D_{i,i} = 0,\ B_{i,i} = 0 \tag{1d}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), E_i = -\widetilde{\Phi}_{,i},\ H_i = -\widetilde{\Psi}_{,i} \tag{1e}$$

where σ_{ij}, ε_{ij}, u_i are the components of stress, strain, and displacement of the nanobeam, respectively; D_i, B_i, E_i, H_i, are the components of electric displacement, magnetic induction, elastic field and magnetic field, respectively; c_{ijkl}, e_{mij}, q_{nij}, s_{im}, d_{ij} and χ_{ij} denote tensors of elastic, piezoelectric, piezomagnetic, dielectric permittivity, magnetoelectric and magnetic permeability constants, respectively. $\widetilde{\Phi}$ and $\widetilde{\Psi}$ are, respectively, the electric and magnetic potentials; ρ is the mass density; $\alpha(|x-x'|,\tau)$ stands for the nonlocal modulus, which can be acted as an attenuation function incorporating the Euclidean form distance $|x-x'|$ and the scale coefficient τ. Here, the coefficient τ is expressed as $\tau = e_0a/l$, where e_0 is a constant determined by comparing the plane wave dispersion curves with those obtained from the atomistic lattice dynamics; a and l are, respectively, the internal and external characteristic lengths which are related to the lattice parameter, granular size, crack length and wavelength of the nanostructure.

In most engineering applications, the integral form of constitutive relations is difficult to solve analytically. Thus, an equivalent differential form of the nonlocal elasticity constitutive (1) can be approximated as

$$[1-(e_0a)^2\nabla^2]\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{mij}E_m - q_{nij}H_n \tag{2a}$$

$$[1-(e_0a)^2\nabla^2]D_i = e_{ikl}\varepsilon_{kl} + s_{im}E_m + d_{in}H_n \tag{2b}$$

$$[1-(e_0a)^2\nabla^2]B_i = q_{ikl}\varepsilon_{kl} + d_{im}E_m + \chi_{in}H_n \tag{2c}$$

where ∇^2 denotes the Laplace operator and e_0a represents the scale coefficient of nanostructures. For one dimensional Euler-Bernoulli nanobeam, Eq. (2) can be rewritten as the following form

$$\sigma_{xx} - (e_0a)^2\frac{\partial^2\sigma_{xx}}{\partial x^2} = \widetilde{c}_{11}\varepsilon_{xx} - \widetilde{e}_{31}E_z - \widetilde{q}_{31}H_z \tag{3a}$$

$$D_x - (e_0a)^2\frac{\partial^2 D_x}{\partial x^2} = \widetilde{s}_{11}E_x + \widetilde{d}_{11}H_x \tag{3b}$$

$$D_z - (e_0a)^2\frac{\partial^2 D_z}{\partial x^2} = \widetilde{e}_{31}\varepsilon_{xx} + \widetilde{s}_{33}E_z + \widetilde{d}_{33}H_z \tag{3c}$$

$$B_x - (e_0a)^2\frac{\partial^2 B_x}{\partial x^2} = \widetilde{d}_{11}E_x + \widetilde{\chi}_{11}H_x \tag{3d}$$

$$B_z - (e_0a)^2\frac{\partial^2 B_z}{\partial x^2} = \widetilde{q}_{31}\varepsilon_{xx} + \widetilde{d}_{33}E_z + \widetilde{\chi}_{33}H_z \tag{3e}$$

where $\widetilde{c}_{ij}$, $\widetilde{e}_{ij}$, $\widetilde{q}_{ij}$, $\widetilde{s}_{ij}$, $\widetilde{d}_{ij}$ and $\widetilde{\chi}_{ij}$ stand for the reduced constants of nanobeam under the plane stress state, which can be separately expressed as follows