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On the delay bound for coordination of multiple generic linear agents under arbitrary topology with time delay

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ABSTRACT

A coordination control of multiple generic linear homogeneous agents under arbitrary network topology with uniform and fixed time delay is proposed in this paper. From the network topology, the agents are categorized into two groups: those within the closed strong components (group 1) and those outside the closed strong components (group 2). It is shown that under allowable delay bound, the agents of group 1 reach synchronization while the agents of group 2 converge asymptotically to the convex hull spanned by the synchronized agents of group 1. The technique of semi-discretization is applied for computing the delay bound. For a specific time delay, the method is also feasible in finding an optimal coordination control gain with the fastest decay rate. A linear matrix inequality method is also given to show an alternative way to find the maximum allowable delay bound. An illustrative simulation is presented to validate our obtained theoretical results.

1. Introduction

Encouraged by the development of cooperative control such as UAV cooperative decision and control, formation control for multi-robot teams, flocking behavior of birds, fish, insects and land animals, distributed autonomous sensors and actuator networks, a lot of scholars have engaged in research of distributed coordination in multi-agent systems these years [1,3,21,33–37]. Protocols that achieve agreement and solve consensus problems are designed for decentralized control of multi-agent systems, which is one of the most popular issues. Plenty of theoretical and practical results on consensus control have appeared in the literature [4–7,18,19,22–24]. The communication time delay existing in multi-agent system involved in consensus control has attracted generous studies on the convergence analysis and performance appraisal due to the effect of the delay. Along this research direction, considerable development has been made [8,9], while most of which only deal with multi-agent systems with integrator dynamics. The synchronization problem of linearly coupled ordinary differential equations (ODEs) was solved and a common solution was derived in [10]. In [11], further investigation was made into the synchronization problem of a group of delayed homogenous linear multi-agent systems. Leaderless and leader-following consensus approach was made in the time-domain and frequency-domain (Lyapunov theorems and the Nyquist criterion) for multi-agent systems (MASs) under a directed network topology with communication and input time delays in [12]. Lyapunov and linear matrix inequality (LMI) method are applied in [13] for the global synchronization of a set of linearly hybrid coupled systems with time-varying time delay.

Meanwhile, the coordination control problem of generic linear multi-agent systems has become one of research hot spots recently. In such systems, each agent modeled as generic linear component is able to deal with the practical application with more complex context. Due to the additional complication caused by the dynamics of each individual linear agent, the relevant coordination control design of generic linear MASs is far more difficult than those with integrator dynamics. Synchronization and containment control study of generic linear multi-agent system without time delay are reported in [14–17]. In [20], a framework of the general case with arbitrary network topology was proposed as the very recent extension of the previous results. Coordination performance analysis of linear agents with communication time delay was presented in [25–27,38–40]. Specifically, in [25,26], Lyapunov–Krasovskii functional based LMI method was introduced to solve the maximum allowable time delay bound for the linear agents to achieve synchronization, while [27] derived consensus-based formation control condition for the linear MAS. In [38], by

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exploiting the adaptation of couplings, a distributed adaptive controller is provided and sufficient conditions to ensure synchronization of linear systems subject to multiple communication delays are presented, which are based on both LMI technique and Lyapunov–Krasovskii functional analysis. Synchronization control of mechanical systems subject to heterogeneous communication delay is addressed in [40] using an integral-sliding adaptive controller. In [39], multisynchronization of neural networks subject to time-varying communication delays is investigated, then convergence criterion is derived in terms of LMIs. Different from the above existing literature, a more general framework will be designed here to *realize general coordination*, including convergence and containment behaviors, for the linear agents with arbitrary network topology. Furthermore, the convergence analysis will be performed by the method of semi-discretization and a typical LMI method [28] to handle the time delay.

As a well-known numerical computing technique of stability analysis for structural and fluid mechanics, the semi-discretization method approximates the time delayed entries of the dynamical equation by piecewise constants in each discretization time interval, meanwhile keeping the other terms untouched [29,30]. A mapping of the system state vectors is then constructed in a finite dimensional state space based upon the exact solution of the original dynamics of the linear system in this method. In [29], semi-discretization method was applied for analyzing the stability of delayed non-neutral periodic systems. The results in [29] showed that if the semi-discretization level is high enough (i.e., each step of time interval is small enough), the transformed discretized system will keep the original stability properties. By computing the maximum absolute eigenvalues of the mapping matrix and comparing it with 1, the stability region and stability boundary of the system can be determined. Furthermore, an optimal control design is achieved by minimization of the maximum absolute eigenvalues of the mapping matrix.

In comparison with the LMI method employed in [25,26], the semi-discretization method has the advantage of providing an efficient algorithm to calculate the upper bound of the allowable time delay. What's more, it has a major advantage in designing an optimal feedback control with the fastest decay rate for a given communication time delay. By carefully exploiting the form of the Laplacian matrix, a feedback coordination control protocol is derived from the method of semi-discretization to realize coordination of a group of generic linear agents subject to time delays that are uniform and fixed. The method turns out to be efficient for feedback control design because of its substantially fast convergence rate, as well as the effective calculation of the delay bound. It is worth pointing out that, for the coordination control of the multi-agent systems with fixed time delay, the semi-discretization method usually comes out with a better solution than the LMI method. However, the LMI method has a good advantage that it can be adopted to derive the stability condition for multi-agent systems with time-varying delays while the semi-discretization method is not workable in this regard. For this reason, in the current paper, we also present a typical LMI solution [28] to calculate the maximum allowable delay bound. Although the stability result by the LMI method may be more conservative than the semi-discretization method, it is potentially a good choice for more complicated delay condition, e.g., multi-agent systems with time-varying delay. A brief comparison of the semi-discretization method and the LMI method demonstrates that both are effective solutions to our coordination control problem, with each one having its own distinct characteristics.

This rest of the paper is organized into four sections. Section 2 introduces some graph and matrix theory notions. In Section 3, the delay bound for coordination of multiple generic linear agents under arbitrary topology with time delay is derived employing the semi-discretization method and the LMI method, respectively. Section 4 presents a simulation example to demonstrate the feasibility of the theoretical results. Finally, concluding remarks are made in Section 5.

2. Background and preliminaries

We use a digraph to describe the network topology of a multi-agent system. $G=(V,\varepsilon,A)$ is an Nth order weighted digraph with a node set $V=\{1,2,\dots,N\}$, an edge set $\varepsilon\subset V\times V$, and a weighted adjacency matrix $A=\left[a_{ij}\right]\in\mathbb{R}^{N\times N}$, where a_{ij} represents the weight of the corresponding directed edge. Furthermore, we define the Laplacian matrix $L=\left[l_{ij}\right]$ of G as

$$l_{ij}=\begin{cases}-a_{ij} & j\neq i\\ \sum_{k=1,k\neq i}^{N}a_{ik} & j=i\end{cases}$$

$\mathbf{1}_N=[1\ \ 1\ \ \cdots\ \ 1]^T\in\mathbb{R}^N$ is a right eigenvector of L associated with the zero eigenvalue. Moreover, the zero eigenvalue is a simple eigenvalue of L and every other eigenvalue of L is with a positive real part if and only if there exists a directed spanning tree in G [2].

A digraph G is called strongly connected if there is a directed path between any pair of distinct nodes. A strongly connected component of G is a maximal strongly connected subgraph of G. A closed strong component is the one such that there are no inwardly directed edges from a vertex outside the component to any vertex in the component. More discussions about closed strong components can be found in [20].

3. Main results

Consider a set of linear agents described by

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\quad i=1,\dots,N, \tag{1}$$

where $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $x_i\in\mathbb{R}^n$ is the state variable of agent i, and $u_i\in\mathbb{R}^m$ is the control signal of agent i. Specifically, we introduce a static feedback controller

$$u_i(t)=H\sum_{j=1}^{N}a_{ij}\left(x_j(t-d)-x_i(t-d)\right), \tag{2}$$

where H is a feedback gain matrix of compatible order and d denotes the uniform and fixed communication delay for information exchange of the linear agents.

As is usually treated in the coordination control design problem, (A,B) is assumed to be a stabilizable matrix pair.

Suppose that there are R $(1\le R\le N)$ closed strong components, say $G_1,\dots,G_R$, in graph G, and further the node set of G_r is indexed as follows:

$$V(G_r)=\left\{\sum_{j=0}^{r-1}n_j+1,\dots,\sum_{j=0}^{r}n_j\right\},\ \ 1\le r\le R,$$

where $n_0=0$. Let $V_{\bar{R}}$ be the set of nodes lying outside the closed strong components, i.e.,

$$V_{\bar{R}}=V-\bigcup_{r=1}^{R}V_r.$$

If $V_{\bar{R}}\neq\emptyset$, let $G_{\bar{R}}$ denote the graph induced from G with node set $V_{\bar{R}}$. The Laplacian matrix of G then has the following form:

$$L=\begin{bmatrix}L_1 & \cdots & 0 & 0\\ \vdots & \ddots & \vdots & \vdots\\ 0 & \cdots & L_R & 0\\ F_1 & \cdots & F_R & F\end{bmatrix}, \tag{3}$$