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Parameterized algorithms for conflict-free colorings of graphs ☆

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we study the conflict-free coloring of graphs induced by neighborhoods. A coloring of a graph is conflict-free if every vertex has a uniquely colored vertex in its neighborhood. The conflict-free coloring problem is to color the vertices of a graph using the minimum number of colors such that the coloring is conflict-free. We consider both closed neighborhoods, where the neighborhood of a vertex includes itself, and open neighborhoods, where a vertex does not include in its neighborhood. We study the parameterized complexity of conflict-free closed neighborhood coloring and conflict-free open neighborhood coloring problems. We show that both problems are fixed-parameter tractable (FPT) when parameterized by the cluster vertex deletion number of the input graph. This generalizes the result of Gargano and Rescigno [Theoretical Computer Science, 2015] that conflict-free coloring is FPT parameterized by the vertex cover number. Also, we show that both problems admit an additive constant approximation algorithm when parameterized by the distance to threshold graphs.

We also study the complexity of the problem on special graph classes. For split graphs, we give a polynomial time algorithm for closed neighborhood conflict-free coloring problem, whereas we show that open neighborhood conflict-free coloring is NP-complete. For cographs, we show that conflict-free closed neighborhood coloring can be solved in polynomial time and conflict-free open neighborhood coloring need at most three colors. We show that interval graphs can be conflict-free colored using at most four colors.

1. Introduction

The conflict-free coloring problem was introduced by Even et al. [1] to study the frequency assignment problem for cellular networks. These networks contain two types of nodes, base stations and clients. Fixed frequencies are assigned to base stations to allow connections to clients. Each client scans for the available base stations in this neighborhood and connects to one of the available base stations. Suppose if two base stations are available to a client, which are assigned the same frequency then mutual interference occurs and the connection between the client and base stations can become noisy. Our aim is to reduce the disturbances occur in connections between base stations and clients. The frequency assignment problem on cellular networks is an assignment of frequencies to base stations such that for each client there exists a base station of unique frequency within his region. The goal here is to minimize the number of assigned frequencies, since available frequencies are limited and expensive.

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We can model this problem using the hypergraphs. The vertices of the hypergraph correspond to the base stations and the set of base stations available for each client is represented by a hyperedge. The problem reduces to assigning frequencies to vertices of a hypergraph such that each hyperedge contains a vertex of unique frequency.

Definition 1. Let $\mathcal{H} = (V, E)$ be a hypergraph, a coloring $C_{\mathcal{H}}$ is called conflict-free coloring of $\mathcal{H}$ if for every $e \in E$ there exists a vertex $u \in e$ such that for all $v \in e$, $u \neq v$ we have $C_{\mathcal{H}}(u) \neq C_{\mathcal{H}}(v)$. The minimum number of colors needed to conflict-free color the vertices of a hypergraph $\mathcal{H}$ is called the *conflict-free chromatic number* of $\mathcal{H}$.

Conflict-free coloring is well studied for hypergraphs induced by geometric objects like, intervals [2], rectangles [3], unit disks [4] etc. This problem also has applications in areas like radio frequency identification and robotics, VLSI design and many other fields.

In this paper, we study the conflict-free coloring of hypergraphs induced by graph neighborhoods. Let $G = (V, E)$ be a graph, for a vertex $v \in V(G)$, $N(v)$ denotes the set consisting of all vertices which are adjacent to v, called open neighborhood of v. The set $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood of v. The conflict-free open neighborhood (CF-ON) coloring of a graph G is defined as the conflict-free coloring of the hypergraph $\mathcal{H}$ with

$$V(\mathcal{H}) = V(G) \qquad \text{and} \qquad E(\mathcal{H}) = \{N(v) \,:\, v \in V(G)\}$$

Similarly the conflict-free closed neighborhood (CF-CN) coloring problem can be defined.

Alternatively we can also define both CF-CN coloring and CF-ON coloring problems as follows. Given a graph G and a coloring C_G, we say that a subset $U \subseteq V(G)$ has a *unique color* with respect to C_G if there exists a color c such that $|\{u \in U \mid C_G(u) = c\}| = 1$.

Definition 2.

1. A coloring C_G of a graph G is called *conflict-free closed neighborhood (*CF-CN*) coloring* if for every vertex $v \in V(G)$, the set $N[v]$ has a unique color.
2. A coloring C_G of a graph G is called *conflict-free open neighborhood (*CF-ON*) coloring* if for every vertex $v \in V(G)$, the set $N(v)$ has a unique color.

The minimum value k for which there is a CF-ON (resp. CF-CN) coloring of G with k colors is called the CF-ON *(resp.* CF-CN*) chromatic number* of G and is denoted as $\chi_{cf}(G)$ (resp. $\chi_{cf}[G]$).

Related work. Gargano et al. [5] studied the complexity of conflict-free colorings induced by the graph neighborhoods and showed that the conflict-free closed neighborhood coloring problem is NP-complete when two colors are used. In the parameterized setting, both conflict-free closed and open neighborhood colorings are fixed-parameter tractable (FPT), when parameterized by the vertex cover number or the neighborhood diversity of the graph [5]. Both problems are FPT parameterized by the tree-width, which follows from an application of Courcelle's theorem [6] and the fact that the CF-CN and CF-ON coloring problems can be expressed by a monadic second order (MSO) formula. Ashok et al. [7] showed that maximizing the number of conflict-free colored edges in hypergraphs is FPT when parameterized by the number of conflict-free edges in a solution.

Our contributions. In this paper, we give parameterized algorithms for CF-CN coloring and CF-ON coloring problems with respect to various *distance-to-triviality* [8,9] parameters. They measure how far a graph is from some class of graphs for which the problem is tractable. Then, it is natural to parameterize by the distance of a general instance to a tractable class. The main advantage of studying structural parameters is, if a problem is tractable on a class of graphs $\mathcal{F}$, then it is natural to expect the problem might be tractable on a class of graphs which are close to $\mathcal{F}$. Our notion of *distance* to a graph class is the vertex deletion distance. More precisely, for a class $\mathcal{F}$ of graphs we say that X is an $\mathcal{F}$-modulator of a graph G if there is a subset $X \subseteq V(G)$ such that $G \setminus X \in \mathcal{F}$. If the size of the smallest modulator to $\mathcal{F}$ is k, we also say that the distance of G to the class $\mathcal{F}$ is k.

We study the parameterized complexity of the conflict-free coloring problems with respect to the distance from the following graph classes: *cluster graphs* (disjoint union of complete graphs) and *threshold graphs*. Studying the parameterized complexity of conflict-free coloring problem with respect to these parameters improves our understanding about the tractable parameterizations. For instance, the parameterization by the distance to cluster graphs directly generalizes vertex cover and is not comparable with tree-width (see Fig. 1). In particular, we obtain the following results.

- We show that both variants of conflict-free coloring problems are FPT when parameterized by the size of a modulator to cluster graphs (cluster vertex deletion number).
- We show that the CF-CN (resp. CF-ON) coloring problem admits an additive 1-approximation (resp. 2-approximation) algorithm when parameterized by the size of a modulator to threshold graphs.

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