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Taxi-sharing: Parameterized complexity and approximability of the dial-a-ride problem with money as an incentive

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A B S T R A C T

We study, in this paper, a taxi-sharing problem, called Dial-a-Ride problem with money as an incentive (DARP-M). This problem consists in defining a set of taxis that will be shared by different clients in order to reduce their bill by a given factor $\alpha < 1$. To achieve this, each client shares the cost of the ride with other passengers. More precisely, the fragments of the ride in which the client is alone is fully paid by this client and, for each fragment in which the client shares the taxi with other passengers, the cost is equally divided between the passengers. In addition to this cost constraint, the taxi must satisfy a time window constraint for each passenger and a capacity constraint.
We define three versions of the problem: max-DARP-M where the objective is to drive the maximum number of clients with an arbitrarily large number of taxis; max-1-DARP-M in which we want to drive the maximum number of clients with one taxi; and 1-DARP-M which consists in deciding whether it is possible to drive at least one client while satisfying the constraints. We study the parameterized complexity and approximability of those problems with respect to four parameters: the factor α, the capacity *capa* of the taxis, the maximum size TW of the time windows of the clients, and the value S of an optimal solution.
Among other results, we prove that 1-DARP-M is NP-Complete and max-DARP-M and max-1-DARP-M cannot be approximated in polynomial time to within any variable ratio even if α, *capa* and TW are fixed and if the road network is a planar graph. We also give a polynomial algorithm for max-1-DARP-M for the case where *capa* and TW are fixed and where the network does not contain a circuit. This algorithm implies a $\frac{1}{\sqrt{n}}$-polynomial approximation for max-DARP-M.

1. Introduction

The Dial-a-Ride problem (DARP) consists in the search for an optimal route for many vehicles in order to drive people from their respective origin to their respective destination. This model is used, for example, to determine an optimized route for taxis in order to pick up passengers. We focus in this article on the complexity of a version of this taxi-sharing problem

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in which the price paid by each passenger is shared. Such a version, called Dial-a-Ride problem with money as an incentive, was previously introduced and studied in [20,21].

Ride-sharing, including Taxi-sharing, has been massively studied for the last fifteen years due to the economical impact and the ecological impact of such a research. Indeed, optimizations reducing the number of vehicles or the number of travels is an obvious way to reduce the costs and the greenhouse gas emissions. DARP can be seen as a subproblem of the general pickup and delivery problem (GPDP) described in [22] in which the goal is to transport a resource from different pickup locations to drop off locations. In DARP, we consider a human resource (the clients) and each pickup or drop off location is associated with exactly one client. The consequence of this specific resource is that one must be aware of the user inconvenience.

1.1. Related work on the DARP problem

DARP can hardly be defined as a unique problem. The feasible and optimal solutions of a Dial-a-Ride problem depend on the measure, the fleet parameters and the clients constraints. Thus, the variety of studies about DARP is not surprising.

Considering the measure, one may optimize the vehicle travel cost, see for example [3,16,19], the total travel time [10] or the profit [7]. Another option is to maximize the number of satisfied requests or a combination of all those parameters [20,21,23].

Some constraints modelize the user convenience. A usual option is to search for a feasible solution considering time windows [4,10,19,23] as it has been done for the more general pickup and delivery problem [8]. This last problem is solved with a column generation scheme where columns define admissible routes. In [10,19], the authors develop a similar approach merging a branch-and-cut algorithm with column generation. In [4,23], the problem is solved using a Tabu search heuristic. Another option to modelize the user convenience is to tend to minimize the excess ride time [2,11,14].

Finally one can consider either the static problem in which all the requests are known in advance or the dynamic version in which the requests may occur at any time [1,6,11,20,21], this problem is usually solved using a local search heuristic.

A recent review about the Dial-a-Ride problem and some of its generalizations may be found in [15]. We refer the reader to [5,10] for a more specific review about DARP.

1.2. DARP with Money as an incentive

We focus on a problem where the goal is to find a feasible solution satisfying a client cost constraint. Few papers focused on that constraint. In [20,21], the authors study the version of the problem in which each client, traveling by taxi, may share the cost of the ride with other passengers. More precisely, the fragments of the ride in which the client is alone is fully paid by this client. On the contrary, for each fragment in which the client shares the taxi with other passengers, the cost is equally divided between the passengers. The problem consists in the search for a ride in which every client does not pay more than the cost he would pay alone in a taxi traveling directly from his origin to his destination. Note that a client can be served by being assigned to a private ride but each client must also satisfy a time window constraint. The objective is to maximize the number of served clients. This problem is called Dial-a-Ride problem with Money as Incentive and is denoted by DARP-M.

In [21], the authors give a reduction from the Traveling salesman problem to DARP-M, based on the sole time windows constraint. However, no taxi is shared, all the clients are driven in a private ride. It proves that serving all the clients and satisfying a time windows constraint is NP-Complete. Considering this reduction, DARP-M can be seen as a generalization of TSP in which we add a sharing cost constraint. Although this reduction clearly shows that DARP-M is strongly NP-Complete, it does not reflect the hardness of determining if at least two clients can be served by sharing a taxi while satisfying the cost constraint. That simpler question is not insignificant as it leads to a natural greedy algorithm for DARP-M in which we group clients who can share a taxi until all of them have to be assigned to private rides.

Furthermore, it was shown by [18] that searching for a (not elementary) shortest path between a source and a sink satisfying a time windows constraint is weakly NP-Complete as it can be solved in polynomial time if the width of the time windows is polynomially bounded. Consequently, as the reduction of [21] uses only the time windows constraint and as it is from the strongly NP-Complete problem TSP, it seems that it cannot be easily adapted to prove the hardness of determining if at least two clients can share a taxi.

1.3. Our contributions

We focus on the parameterized complexity and the parameterized approximability of three problems derived from DARP-M defined by [20,21]. The purpose of this paper is mainly to investigate how hard the cost constraint is. Particularly, we point out the fact that every hardness result we give is true even if we do not take into account the time windows.

We now formally define the problems we study. We work in a directed graph $G = (V, A)$. We are given a set of n clients arbitrarily numbered in $[\![1; n]\!]$. A *client* is attached to two nodes which are respectively the origin and the destination of the client. In order to avoid any ambiguity, a node cannot be the origin or the destination of two clients. If two clients books from and/or for the same place, we can simply duplicate the node in the graph. There are, in this paper, two ways to refer to a client and its associated nodes. Either we know the number i of the client and, in that case, we refer to the client