Computers and Mathematics with Applications ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Diversity of exact solutions to a (3+1)-dimensional nonlinear evolution equation and its reduction

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A R T I C L E I N F O

Article history:
Received 12 April 2018
Received in revised form 28 May 2018
Accepted 9 June 2018
Available online xxxx

Keywords:
Test function method
Exact solutions
Symbolic computation

A B S T R A C T

In this paper, a (3+1)-dimensional nonlinear evolution equation and its reduction is studied by use of the Hirota bilinear method and the test function method. With symbolic computation, diversity of exact solutions is obtained by solving the under-determined nonlinear system of algebraic equations for the associated parameters. Finally, analysis and graphical simulation are given to reveal the propagation and dynamical behavior of the solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) play an important role in mathematical physics [1–7]. Various mechanical features in fluid dynamics, optical communications and nonlinear vibration are described by NLEEs [8–11]. Generally speaking, it is very difficult to find exact solutions to NLEEs [12–14]. With the development of symbolic computation, it is reasonable to employ test function method in constructing exact solutions to NLEEs [15,16]. Further, it is of importance to solve high-dimensional NLEEs to study the associated spatiotemporal features [17–23].

In this paper, we will study a $(3+1)$-dimensional NLEE [24–30] as

$$3\,u_{xz} - (2\,u_t + u_{xxx} - 2\,u\,u_x)_y + 2\,(u_x\,\partial_x^{-1}\,u_y)_x = 0, \tag{1}$$

where ∂_x^{-1} stands for an inverse operator of $\partial_x = \frac{\partial}{\partial x}$. Eq. (1) was originally introduced as a model for the study of algebraic-geometrical solutions [24], and its integrability and large classes of exact solutions have been studied, e.g., the soliton, positon, negaton and rational solutions [25–29]. Further, two types of resonant solutions are obtained by the parameterization for wave numbers and frequencies for linear combinations of exponential traveling waves [30].

Through the following transformation

$$u = -3\,[\ln f(x, y, z, t)]_{xx}, \tag{2}$$

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https://doi.org/10.1016/j.camwa.2018.06.020

Eq. (1) can be cast into the bilinear form as

$$(3\,D_x\,D_z - 2\,D_y\,D_t - D_x^3\,D_y)f\cdot f = 0, \tag{3}$$

where $D_x\,D_z$, $D_y\,D_t$ and $D_x^3\,D_y$ are bilinear operators [12] defined by

$$D_x^\alpha D_y^\beta D_z^\gamma D_t^\delta(f\cdot g) = \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^\alpha\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^\beta\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^\gamma\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^\delta \times f(x,y,z,t)g(x',y',z',t')\Big|_{x'=x,y'=y,z'=z,t'=t}.$$

We assume that the solution to Eq. (3) is in the form of

$$f = e^{-\xi} + \delta_1\,\cos(\eta) + \delta_2\,\cosh(\gamma) + \delta_3\,e^{\xi}, \tag{4}$$

or

$$f = e^{-\xi} + \delta_1\,\sin(\eta) + \delta_2\,\sinh(\gamma) + \delta_3\,e^{\xi}, \tag{5}$$

where $\xi = a_1\,x + b_1\,y + c_1\,z + d_1\,t$, $\eta = a_2\,x + b_2\,y + c_2\,z + d_2\,t$, $\gamma = a_3\,x + b_3\,y + c_3\,z + d_3\,t$ and a_i, b_i, c_i, d_i, and δ_i $(i = 1,2,3)$ are some constants to be determined later. Based on Eq. (4) or Eq. (5), we can derive exact solutions to Eq. (1).

The structure of this paper is as follows: In Section 2, we will solve Eq. (3) and obtain the exact solutions to Eq. (1). In Section 3, we will give analysis and discussion on our solutions. Some figures describing the characteristics of our solutions will be presented. In Section 4, we will conclude our results.

2. Diversity of exact solutions

2.1. Case I: based on the test function (4)

Substituting Eq. (4) into Eq. (3), we can obtain a large expression in terms of $\cos(\eta)e^{\xi}$, $\cos(\eta)e^{-\xi}$, $\sin(\eta)e^{\xi}$, $\sin(\eta)e^{-\xi}$, $\cosh(\gamma)e^{\xi}$, $\sinh(\gamma)e^{\xi}$, $\cosh(\gamma)e^{-\xi}$, $\sinh(\gamma)e^{-\xi}$, $\cos(\eta)\cosh(\gamma)$, $\sin(\eta)\sinh(\gamma)$, etc., which generate a list of algebraic equations as

$$\begin{cases} 3a_1c_1 - 3a_2c_2 - 2b_1d_1 + 2b_2d_2 - a_1^3b_1 - a_2^3b_2 + 3a_1^2a_2b_2 + 3a_1a_2^2b_1 = 0,\\ 3a_1c_1 + 3a_3c_3 - 2b_1d_1 - 2b_3d_3 - a_1^3b_1 - a_3^3b_3 - 3a_1^2a_3b_3 - 3a_1a_3^2b_1 = 0,\\ 3a_1c_2 + 3a_2c_1 - 2b_1d_2 - 2b_2d_1 - a_1^3b_2 + a_2^3b_1 - 3a_1^2a_2b_1 + 3a_1a_2^2b_2 = 0,\\ 3a_1c_3 + 3a_3c_1 - 2b_1d_3 - 2b_3d_1 - a_1^3b_3 - a_3^3b_1 - 3a_1^2a_3b_1 - 3a_1a_3^2b_3 = 0,\\ 3a_3c_3 - 3a_2c_2 - 2b_3d_3 + 2_b2d_2 - a_2^3b_2 - a_3^3b_3 + 3a_2^2a_3b_3 + 3a_2a_3^2b_2 = 0,\\ 3a_2c_3 + 3a_3c_2 - 2b_2d_3 - 2b_3d_2 + a_2^3b_3 - a_3^3b_2 + 3a_2^2a_3b_2 - 3a_2a_3^2b_3 = 0,\\ \delta_3(12a_1c_1 - 8b_1d_1 - 16a_1^3b_1) + \delta_2^2(3a_3c_3 - 2b_3d_3 - 4a_3^3b_3) + \delta_1^2(2b_2d2 - 3a_2c_2 - 4a_2^3b_2) = 0. \end{cases} \tag{6}$$

With symbolic computation, we solve two sets of parameters from Eq. (6):

The first set in this case is

$$\Big\{a_1 = 1, a_2 = 0, b_1 = -\frac{4}{3a_3^2}, b_2 = -2, b_3 = -\frac{4}{3a_3}, c_1 = -\frac{8(2a_3^3+1)}{9a_3}, c_2 = -\frac{2(3a_3^4+6a_3-4)}{9a_3^2}, c_3 = -\frac{4(a_3^2+3a_3+2)}{9a_3^2}, d_1 = \frac{a_3^3-a_3+2}{2a_3}, d_2 = -1, d_3 = 1, \delta_3 = \frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4}\Big\}, \tag{7}$$

where a_3, δ_1 and δ_2 are real constants.

Substituting parameters of Eq. (7) into Eq. (4), we have $f = e^{-A_1} + \delta_1\,\cos(B_1) + \delta_2\,\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}$, which leads to the exact solutions to Eq. (1) as

$$u = -3\frac{e^{-A_1} + \delta_2a_3^2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}}{e^{-A_1} + \delta_1\cos(B_1) + \delta_2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}} + 3\frac{(-e^{-A_1} + a_3\delta_2\sinh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1})^2}{(e^{-A_1} + \delta_1\cos(B_1) + \delta_2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1})^2} \tag{8}$$

Please cite this article in press as: Y.-H. Yin, et al., Diversity of exact solutions to a (3+1)-dimensional nonlinear evolution equation and its reduction, Computers and Mathematics with Applications (2018), https://doi.org/10.1016/j.camwa.2018.06.020.