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Determining the optimal parameters for the MHD flow and heat transfer with variable viscosity and Hall effect

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ABSTRACT

The direct and optimal control solution of the laminar, fully developed, steady MHD flow of an incompressible, electrically conducting fluid in a duct is considered together with the heat transfer. The flow is driven by a constant pressure gradient and an external uniform magnetic field. The fluid viscosity is temperature dependent varying exponentially and the Hall effect, viscous and Joule dissipations are taken into consideration. The control problem is solved by the *discretize-then-optimize* approach using mixed finite element method for the MHD and energy equations. The control formulations with the Hall and viscosity parameters, the Hartmann and Brinkmann number are given to regain the desired velocity and temperature of the MHD flow.

1. Introduction

The problem of laminar flow and the heat transfer through rectangular channels is of great importance in designing heat exchangers, cooling of electronic systems, chemical reactors, nuclear reactor and combustion systems. Some researchers have studied the magnetic effects on the flow and heat transfer of electrically conducting fluids in rectangular ducts by using numerical methods. Türk and Tezer-Sezgin [1] have given a solution of natural convection flow in square enclosures under magnetic field using the finite element method (FEM). Alsoy-Akgün and Tezer-Sezgin [2] have solved natural convection MHD flow equations in a cavity by using both the dual reciprocity boundary element method (DRBEM) and the differential quadrature method (DQM) comparing the solutions from the two methods. DRBEM solution of MHD flow with magnetic induction and heat transfer has been shown by Pekmen and Tezer-Sezgin [3]. A finite difference (FD) solution has been provided for MHD flow free convection flow in a vertical rectangular duct considering the effects of Ohmic heating and viscous dissipation by Umavathi et al. [4]. Kishan and Shekar [5] have showed the combined effects of viscous and Ohmic dissipations on MHD flow by using the FEM. When the viscosity of the fluid is temperature dependent, a significant heat transfer enhancement is achieved even neglecting the Hall effect, viscous and Joule dissipations. Under strong external magnetic field the Hall current is important and has an effect on the current density due to the influence of the electromagnetic force. Attia [6,7] has employed FD solutions to transient MHD flow with heat transfer for dusty fluid with temperature dependent viscosity. The effects of variable viscosity and the magnetic field on the flow and heat transfer of both the fluid and dust particles are shown between parallel plates. The problem of MHD flow and heat transfer with variable viscosity for Newtonian fluids in a rectangular duct with the Hall effect has been investigated using finite difference

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method (FDM) by Sayed-Ahmed and Attia [8]. Sayed-Ahmed [9] has investigated numerically with FDM also the effect of Hall current on MHD flow and heat transfer for Bingham fluids in a rectangular duct. The solution to transient MHD flow and heat transfer of a dusty fluid between parallel plates has been given by Türk and Tezer-Sezgin [10] using Chebyshev spectral collocation when the fluid possesses time-dependent viscosity.

In real world applications involving MHD flow, it is important to be able to control the state of the physical problem under investigation. Therefore, many researchers have studied and derived theoretical results on control problems in fluid mechanics. Abergel and Temam [11] have given the proof of an existence of solutions to control problems for various physical situations, such as distributed control, boundary control in a channel; and they have provided basic numerical algorithms, such as steepest descent and conjugate gradient methods. The boundary control of an electrically conducting fluid has been studied by Hou and Meir [12] using Lagrange multiplier technique as well. Ito and Ravindran [13] have used the thermal convection on part of the boundary and provided a first-order necessary optimality condition for control of cavity and channel type flows. An analysis and discretization of an optimal control problem of the tracking of the velocity and the magnetic field of viscous, incompressible, electrically conducting fluid for the time-periodic MHD equations has been studied by Gunzburger and Trencha [14]. Griesse and Kunisch [15] have considered the control mechanisms by external and injected currents and magnetic fields and provided optimality conditions for a stationary MHD system in a velocity–current formulation. A comprehensive study on the boundary control of the incompressible MHD equations has been conducted by Bornia [16], where a new boundary control approach is proposed based on lifting functions of the boundary condition. Geometric variables have also been considered as control parameters for the energy and cost optimization of the heat exchange by Najafi et al. [17] within the multi-objective optimization using Genetic algorithm. Shape optimization of heat transfer processes has been studied by Hajmohammadi [18,19,20]. Optimal control problem of non-isothermal viscous fluid with a temperature dependent viscosity has been solved by Cox et al. [21] using FEM for the state and adjoint equations and within an *optimize-then-discretize* approach. Recently, Ren et al. [22] have worked on the control problem in 1D MHD flow by a *discretize-then-optimize* approach using SQP optimization algorithm.

In this paper, hydrodynamically and thermally fully-developed, steady MHD flow with heat transfer is studied for an incompressible, electrically conducting variable viscosity fluid in a square duct. The Hall effect, viscous and Joule dissipations terms are included to the governing equations while the induced magnetic field is neglected due to the small Reynolds number. First, the coupled non-linear set of momentum and energy equations are solved by using the mixed FEM for obtaining the velocity of the fluid and the induced magnetic field numerically. The temperature and velocity profiles are depicted for various Hartmann number, viscous and Hall parameters and Brinkman number. It is found that the viscosity parameter has a significant effect on the flow for small values of Hartmann number (Ha) and high values of Hall parameter (m). An increase in Ha decreases the magnitude of velocity for all values of m. Both the product of friction factor (f) and Reynolds number (Re), and Nusselt number (Nu) increase when the viscosity parameter increases for any Ha. The FEM captures the well-known behavior of the MHD flow and heat transfer and show the influences of Hartmann number and Hall parameter in addition to variations in viscosity on the velocity and temperature distributions.

Furthermore, we investigate the problem of controlling the steady flow of a viscous, incompressible, electrically conducting fluid in the duct at a desired velocity and temperature by using the physical parameters of the problem as *control variables*: Hartmann number, Brinkman number, viscosity parameter and Hall parameter. To the best of our knowledge, the control problem for the MHD flow and heat transfer with variable viscosity and the Hall effect in 2D has not been studied. The cost of these control variables as well as the cost of achieving the desired velocity and temperature is added to the objective function(al) that is to be minimized subject to the governing coupled PDEs for the hydrodynamically and thermally fully developed flow in the duct. The first-order necessary optimality conditions are derived from the discretized system of equations obtained by the mixed finite element approximations. The optimization is performed with a gradient-based technique, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. The numerical results confirm that the formulation is well established and the numerical scheme is quite efficient.

2. Problem statement

We consider the optimal control of the laminar, steady flow of a viscous, incompressible, electrically conducting fluid in a long channel of square cross-section (duct) together with heat transfer. A uniform magnetic field of intensity B_0 is applied with an angle to the duct which is perpendicular to the axis of the channel (z-axis). A constant pressure gradient, $-\frac{dp}{dz}$, is applied in the z-direction and the induced magnetic field is neglected due to the assumption of small magnetic Reynolds number. Both the flow and the temperature are steady and fully developed along the channel. The viscosity of the fluid is assumed to vary exponentially with the temperature, however, the Joule and viscous dissipations are not neglected. Due to the strong effect of magnetic force, the Hall effect is necessarily considered as well. The flow is only in the channel axis direction with the velocity $\vec{\mathbf{v}} = \omega(\hat{\mathbf{k}})$ which varies in the duct, i.e. $\omega = \omega(x, y)$ in $\Omega = [0, 1] \times [0, 1]$. The configuration of the problem is shown in Fig. 1. In the following, we investigate the two major ingredients of the problem: (i) mathematical formulation of the physical problem, and (ii) controlling the fluid flow within the duct using the parameters of the problem.

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