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Time-dependent asymptotic behavior of the solution for wave equations with linear memory

Qiaozhen Ma *, Jing Wang, Tingting Liu

School of Mathematics and Statistics, Northwest Normal University, Lanzhou, Gansu 730070, China

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In this article, we consider the long-time behavior of solutions for the wave equation with linear memory. Within the theory of process on time-dependent spaces, we investigate the existence of the time-dependent attractor by using the operator decomposition technique and compactness of translation theorem and more detailed estimates. Furthermore, the asymptotic structure of time-dependent attractor, which converges to the attractor of parabolic equation with memory, is proved. Besides, we obtain a further regular result about u_t.

1. Introduction

Let Ω be an open bounded set of $\mathbb{R}^3$ with smooth boundary $\partial\Omega$. For any $\tau \in \mathbb{R}$, we consider the following equations

$$\begin{cases} \varepsilon(t)u_{tt} + \alpha u_t - \beta\Delta u - \displaystyle\int_0^\infty \mu(s)\Delta\eta^t(s)\mathrm{d}s + f(u) = g(x), \quad \text{in } \Omega \times (\tau, \ \infty), \\ u(x, t) = 0, \quad x \in \partial\Omega, \ t \in \mathbb{R}, \\ u(x, t) = u_0(x, t), u_t(x, t) = \partial_t u_0(x, t), \ x \in \Omega, \ t \le \tau, \end{cases} \tag{1.1}$$

where $u = u(x, t) : \Omega \times [\tau, \infty) \to \mathbb{R}$ is an unknown function, and $u_0 : \Omega \times (-\infty, \tau] \to \mathbb{R}$ is a given past history of u, $g(\cdot) \in L^2(\Omega)$ is independent of time, μ is a summable positive function. $\eta = \eta^t(x, s) := u(x, t) - u(x, t - s)$, $s \in \mathbb{R}^+$. $\varepsilon \in C^1(\mathbb{R})$ is a decreasing bounded function and satisfies

$$\lim_{t\to+\infty} \varepsilon(t) = 0. \tag{1.2}$$

In particular, there exists $L > 0$, such that

$$\sup_{t\in\mathbb{R}}[|\varepsilon(t)| + |\varepsilon'(t)|] \le L. \tag{1.3}$$

The nonlinear term $f \in C^1(\mathbb{R})$, $f(0) = 0$, and for some $C \ge 0$ satisfies

$$|f'(s)| \le C(1 + |s|^2), \quad \forall s \in \mathbb{R}, \tag{1.4}$$

* Corresponding author.
E-mail addresses: maqzh@nwnu.edu.cn (Q. Ma), wj414029715@163.com (J. Wang), lttnwnu91@126.com (T. Liu).

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along with the dissipation condition

$$\liminf_{|s|\to\infty}\frac{f(s)}{s} > -\lambda_1, \quad \forall s\in\mathbb{R}, \tag{1.5}$$

where λ_1 is the first eigenvalue of the strictly positive operator $A=-\Delta$. With respect to the memory component, as in [1,2], we assume that

$$\mu\in C^1(\mathbb{R}^+)\cap L^1(\mathbb{R}^+), \quad \int_0^\infty \mu(s)\mathrm{d}s = m_0 < \infty, \tag{1.6}$$

$$\mu'(s)\le -\rho\mu(s)\le 0, \quad \forall s\ge 0, \tag{1.7}$$

where ρ is a positive constant.

The problem (1.1) can be viewed as a description of viscoelastic solids with fading memory and dissipation due to the viscous resistance of the surrounding medium, as well as of composite materials, phase-fields, and wave phenomena [3,4].

When μ is a Dirac measure at some fixed time instant or when it vanishes, Eq. (1.1) reduces to the damped wave equation, which has been investigated extensively by many authors. For instance, in the case that ε is a positive constant independent of time, the long-time behavior of the solution can be well characterized by using the concept of global attractors in the framework of semigroup. The existence and regular properties of the global attractor have been studied in [5–8]. When ε is a positive constant independent of time and the forcing term g depends on time, the system is a non-autonomous wave equation, the long-time behavior of the solution can be understood in the framework of process, please refer to [6,9–12] for some specific results involving the uniform attractor (or pullback attractors) about non-autonomous case.

When ε is only a positive constant in (1.1), Conti and Pata [13], Borni and Pata [14], Pata and Zucchi [15] investigated the existence of global attractors about the hyperbolic equation with linear memory; besides, Sun, Cao and Duan [16] obtained the existence and asymptotic regularity of the uniform attractor about the non-autonomous system with strong damping, as well as Kloden, Real and Sun [17] scrutinized further the robust exponential attractors to this problem.

However, provided that ε depends explicitly on time in (1.1), such as a positive decreasing function of time $\varepsilon(t)$ vanishing at infinity, leading to time-dependent terms at functional level, these problems become more complex and interesting, because the corresponding dynamical system is still understood within the non-autonomous framework even the forcing term is independent of time, and the classical theory generally fails to capture the dissipation mechanism of the system, as mentioned in [18,19].

To circumvent this issues, in [18], Conti, Pata and Temam presented a notion of time-dependent attractor exploiting the minimality with respect to the pullback attraction property, and constructed a sufficient condition proving the existence of time-dependent attractor based on the theory established by Plinio, Duane and Temam [19]. Meanwhile, within the new framework, the authors studied the following weak damped wave equations with time-dependent speed of propagation

$$\varepsilon(t)u_{tt}+\alpha u_t-\Delta u+f(u)=g(x). \tag{1.8}$$

Besides, they proved that the time-dependent global attractor of (1.8) converged in a suitable sense to the attractor of the parabolic equation $\alpha u_t-\Delta u+f(u)=g(x)$ when $\varepsilon(t)\to 0$ as $t\to+\infty$ [20]. Successively, in [21], they continued to show the existence of an invariant time-dependent global attractor to the following specific one-dimensional wave equation $\varepsilon(t)u_{tt}-u_{xx}+[1+\varepsilon f'(u)]u_t+f(u)=h$, which converges in suitable sense to the classical Fourier equation.

Recently, Meng et al. investigated the long-time behavior of the solution for the wave equation with nonlinear damping $g(u_t)$ on the time-dependent space, in which they found a new technical method verifying compactness of the process via defining the contractive functions, see [22]. In [16], Meng and Liu also showed the necessary and sufficient conditions of the existence of time-dependent global attractor borrowed from the ideas in [5]. Liu and Ma [23] studied the existence of the pullback attractors for the plate equation with time-dependent forcing term on the strong time-dependent Hilbert space.

As we know, in the study of the long-time behavior, especially for attractors, obtaining certain asymptotic compactness for the solution operator is a key step. However, if the equation contains the history memory, just for our problem (1.1), it makes impossible to utilize $(I-P_m)u$ as the test function to capture the asymptotic compactness of the solution process, that is to say, the methods introduced in [5,16] is out of action to our problem. For our purpose, we firstly construct a relatively complicated triple solution space by introducing a new variable. Secondly, we capitalize the decomposition techniques alone with compactness translation theorem to conquer the barriers induced by the critical nonlinearity and the history memory, and then achieve the existence and regularity as well as the upper semi-continuity of time-dependent global attractor for (1.1); in addition, we further prove a regular result of u_t.

It is worth mentioning that the dissipative condition (1.5) is weaker than one in [18,22], indeed, for simplicity, in where the authors made use of the dissipative condition like $\liminf_{|s|\to\infty} f'(s) > -\lambda_1$.

For convenience, hereafter, C (or c) denotes an arbitrary positive constant which may be different from line to line even in the same line.

The rest of this article consists of five sections. In the next section, we define some function sets and iterate some useful lemmas. In Section 3, the existence and regularity of the time-dependent global attractor is obtained. In Section 4, we prove the upper-semicontinuous property of the time-dependent global attractor. Finally, in Section 5, we achieve the further regularity of attractors.