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Dynamic behaviors of interaction solutions of (3+1)-dimensional Shallow Water wave equation☆

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ABSTRACT

Rational solutions and interaction solutions of the (3+1)-dimensional Shallow Water wave equation are obtained depending on N-soliton solution by the Hirota bilinear method and long wave limit method. And breather, lump solutions and line rogue waves are obtained from two-soliton solution as well as their interaction solutions are derived from four-soliton solution. After the collision, breathers and lumps remain unchanged, but the line rogue wave has a difference. Furthermore, three-soliton solution is discussed, the interactions between some hybrid solutions, such as hybrid solution between the stripe soliton and breather, hybrid solution between stripe soliton and lump solution and so on, are demonstrated in detail. More interestingly, stripe soliton remain unchanged, but the amplitudes of the periodic line waves, lump solutions and rogue waves have a great change after the collision.

1. Introduction

The study for integrability to nonlinear evolution equation is always a hot topic in soliton theory and integrable systems, which is viewed as a key step of exact solvability. Much work of integrable systems are studied, such as, Lax pair [1–3], Hamiltonian structure [4–7], infinite conservation laws [8–11], Bäcklund transformation [12–15], bilinear integrability [16–19]. Many kinds of solutions have been obtained, such as rational solutions [20–22], lump solutions [23–26] and so on, by the bilinear methods. Rogue wave solution [27–29] is a kind of rational solutions which is localized in both space and time and observed in the deep ocean firstly. The amplitude of Rogue wave's wave could reach up to 20-30m and appear from nowhere and disappear without trace. Furthermore, its destructive force is so strong that the unexpected disaster in the world is caused. In recent years, the rogue wave phenomenon has appeared in the category of social and scientific contexts, ranging from hydrodynamics [30] and geophysics to oceanography, financial markets [31,32], Bose–Einstein condensation, nonlinear optics [33,34], and plasma physics [35]. In 1983, the first order or fundamental rogue waves of the nonlinear Schrödinger equation was first obtained by Peregrine in 1983 [36], and higher-order rogue waves of the NLS equation are presented recently. Lately, there are also lots of complex systems that possess rogue wave solutions, such as the Hirota equation [28], AB system [29], KP equation [37,38], the Sasa–Satsuma equation [39], Boussinesq equation [40]. It is interesting that the study on the rogue waves varies from rational solutions to interaction solutions [41,42]. Whatever, the interaction solutions show a series of significant and complicated dynamic behavior, including breathers [41], bright–dark rogue wave pair or rogue

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waves interacting with solitons [42]. And recently, Ma et al. study on mixed lumpkink solutions and interaction solutions made by the quadratic function method [43–46].

In this paper, we pay attention to the (3+1)-dimensional Shallow Water wave equation [22,47]

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

which has been discussed through variable methods, such as rational solution [22], Pfaffian solutions and Grammian solutions [47]. The rogue waves and interaction solutions of the (3+1)-dimensional Shallow Water wave equation have not been studied, but it is important to find analytical form of the rogue waves for boosting the possible applications in ocean research and other fields. In this article, the primary purpose is to study the interaction solutions of (3+1)-dimensional Shallow Water wave equation, such as the interaction between lump solution and rogue wave, the interaction between stripe soliton and other local waves.

The structure of the article is as follows: in section 2, the period line wave, breather and their interactions to (3+1)-dimensional Shallow Water wave equation is presented by the parameter perturbation method and their typical dynamics are analyzed and illustrated. In section 3, lump solution, line rogue wave and three kinds of cases of interaction have been obtained by long wave limit, and the amplitude of rogue wave has a change after the interaction. In section 4, the interactions between stripe soliton and other local waves, which include the soliton and breather, the soliton and lump solution, the soliton and line rogue wave, are discussed and the dynamics behaviors of the interactions are expressed.

2. The period line wave, breather and their interactions

The operator D is the classic Hirota bilinear operator defined as

$$\begin{aligned} &P(D_x, D_y, D_t)F(x, y, t\ldots)G(x, y, t\ldots) \\ &= P(\partial_x - \partial_x', \partial_y - \partial_y', \partial_t - \partial_t', \ldots)F(x, y, t, \ldots)G(x', y', t', \ldots)|_{x'=x, y'=y' t'=t} \end{aligned}$$

where P is a polynomial of $D_x, D_y, D_t, \ldots$.

Eq. (1) can be turned into its bilinear form as

$$(D_x^3 D_y + D_y D_t - D_x D_z) f \cdot f = 0, \tag{2}$$

by the method of variable transformation

$$u = 2(\ln f)_x,$$

where f is a real function about x, y, z, t.

The two-soliton solution of the Shallow Water wave equation through the parameter perturbation method can be rewritten as follows

$$u = 2(\ln f)_x,$$

where

$$f = 1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2}, \tag{3}$$

$$\begin{cases} A_{12} = \dfrac{3q_1 q_2 (p_1 - p_2)(p_2 q_2 - p_1 q_1) + (q_1 - q_2)(q_2 m_1 - q_1 m_2)}{-3q_1 q_2 (p_1 + p_2)(p_2 q_2 + p_1 q_1) + (q_1 - q_2)(q_2 m_1 - q_1 m_2)}, \\ \eta_i = p_i (x + q_i y + m_i z + k_i t) + \eta_i^0, \\ k_i = \dfrac{p_i^2 q_i - m_i}{q_i}, (i = 1, 2). \end{cases}$$

The breather to the Shallow Water wave equation can be presented by choosing

$$p_1 = p_2^* = a_1 + Ib_1, q_1 = q_2^* = a_2 + Ib_2, m_1 = m_2^* = a_3 + Ib_3, \eta_1^0 = \eta_2^{0*}, \tag{4}$$

where $*$ means the conjugate operator and $I^2 = -1$. Without loss of generality, the corresponding function f can be written as

$$f = 1 + 2(\cosh(t + y) - \sinh(t + y))\cos(x + y + 2z - 2t) + 4(\cosh(2t + 2y) - \sinh(2t + 2y)), \tag{5}$$

by taking

$$q_1 = 1 + I, q_2 = 1 - I, p_1 = I, p_2 = -I, m_1 = 2, m_2 = 2.$$

From the expression of function f, we know that the spatial variables x and z are similar, but the spatial variable y and the spatial variables x and z are different in essence. The corresponding dynamics properties to solution u are depicted in Figs. 1 and 2.