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Instability of smoothed particle hydrodynamics applied to Poiseuille flows

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ABSTRACT

Smoothed particle hydrodynamics (SPH) has been widely applied to flows with free surface, multi-phase flow, and systems with complex boundary geometry. However, it has been shown that SPH suffers from transverse instability when applied to simple wall-bounded shear flows such as Poiseuille and Couette flows at moderate and high Reynolds number, Re $\gtrsim$ 1, casting the application of SPH to practical situations into doubt, where the Reynolds number is frequently large. Here, we consider Poiseuille flows for a wide range of Reynolds number and find that the documented instability of SPH can be avoided by using appropriate ratio of smoothing length to particle spacing in combination with a density re-initialization technique, which has not been systematically investigated in simulations of simple shear flows. We also probe the source of the instability and point out the limitations of SPH for wall-bounded shear flows at high Reynolds number.

1. Introduction

Originally, *smoothed particle hydrodynamics* (SPH) was proposed to solve astrophysical problems [1,2]; however, by now there is a much wider range of applications. Because of its meshless and particle-based nature, it is frequently applied to fluid flows with free surfaces [3–6], multi-phase systems [4,3,7,8], and systems with complex boundary geometry [9]. Very recently it was also applied to particle laden flow by fully resolving the flow around moving solid particles [10,11] and wall-bounded turbulence [12]. For a comprehensive review on the fundamentals and applications of SPH see [13–15]. Within the SPH paradigm, there are two different approaches to handle incompressible flow problems, namely *incompressible SPH* (ISPH) which imposes incompressibility by solving the pressure Poisson equation [4,16–20] and *weekly compressible SPH* (WCSPH), which exploits an equation of state to relate density and pressure and approximately imposes the incompressibility by assigning high speed of sound. Mainly because of its pure Lagrangian nature and computational simplicity, WCSPH was extensively used for various flow simulations, e.g. [3,4,9,21,10,11,5,6,12]. In this paper, we refer to WCSPH as SPH and focus on its application to incompressible wall-bounded shear flow.

For shear flow at low Reynolds numbers, SPH yields satisfactory results [9,10,21–23]. However, for larger Reynolds numbers, SPH fails, in particular in simulations of simple shear flow [24,22,25,23]. Imaeda & Inutsuka [24] argued that in standard SPH the particle velocity cannot exactly represent the fluid velocity, therefore, density error gradually increases and invalidates the simulation results. Similar to a recent transport-velocity formulation for SPH [8], the solution provided

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in [24] relies on two velocities, i.e. the particle and fluid velocities. However, the Lagrangian property of SPH is lost in those formulations.

For plane Poiseuille flow, Basa et al. [25] investigated the performance of various viscous force formulations, boundary condition implementations, and particle inconsistency corrections in SPH and observed that the failure of the method cannot be avoided even at a very low Reynolds number, Re $\approx$ 1. The authors identified the inherent inability of SPH to suppress transverse fluctuations as the source of the failure, in agreement with earlier studies [22,26], and termed the instability as *transverse instability*. The same source of failure was found by Meister et al. [23] who considered plane Poiseuille and Couette flows. Although they showed a convergence for small Reynolds number, the numerical error was still considerable ($\approx$ 10% for Re=65). These findings also suggested that the specific choice of kernel function is irrelevant to the failure of the method, as the same type of instability occurs with different kernel functions (B-spline kernel in [25] and quintic spline kernel in [23]). Besides, it is also concluded by these authors that regular initial particle distributions are intrinsically unstable with respect to transverse fluctuations in Poiseuille and Couette flows [23].

The appearance of the instability poses fundamental limitation of the application of SPH to wall-bounded shear flows. We identify the sensitivity of the particle discretization accuracy on particle distribution as the source of the instability and propose strategies to achieve a satisfactory performance of classical SPH simulations. In this paper we also systematically study the effects of method parameters, background pressure, initial particle configurations, and a density re-initialization technique on the performance of SPH for simple shear flow and pipe flow at Reynolds numbers between Re $\approx$ 0.01 and 100.

2. SPH methods

SPH is a Lagrangian approach to solve the Navier–Stokes equations numerically using discrete quasi-particles. The discretization scheme is further elaborated in the following sub-sections.

2.1. Continuity equation

The evolution of density can be formulated using the continuity equation

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\vec{v}, \tag{1}$$

where ρ and $\vec{v}$ are the fluid density and velocity, respectively. In the formulation of SPH, Eq. (1) reads

$$\frac{d\rho_a}{dt} = \rho_a\sum_b \frac{m_b}{\rho_b}\left(\vec{v}_a-\vec{v}_b\right)\cdot\nabla_a W_{ab}\,, \tag{2}$$

where ρ_a is the density of particle a, m_b is the mass of particle b and ∇_a denotes the derivative with respect to the position $\vec{r}_a$ of particle a. W is the kernel function, and $W_{ab}\equiv W\left(\vec{r}_a-\vec{r}_b, h\right)$. In this paper, the cubic spline kernel function given in [10] with a compact support is employed, which reads

$$W\left(\vec{r}, h\right) = \frac{1}{4\pi h^3}\begin{cases}(2-q)^3-4(1-q)^3, & 0\le q<1,\\ (2-q)^3, & 1\le q<2,\\ 0, & q\ge 2,\end{cases} \tag{3}$$

where $q\equiv\left|\vec{r}\,\right|/h$. Alternatively, the density can also be directly obtained using

$$\rho_a = \sum_b m_b W_{ab}\,. \tag{4}$$

Both types of density updating schemes, Eqs. (2) and (4), yield similar results [25]; however, usually the former is preferred as it produces smoother density fields in the vicinity of boundaries [10,21]. In this paper, we adopt the continuity equation Eq. (2) together with a density re-initialization technique using Eq. (4) [4,10].

2.2. Momentum equation

The momentum equation reads

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\vec{v} + \vec{f}, \tag{5}$$

where p is the pressure and $\vec{f}$ denotes the external body force density. Regarding discretization, we apply the gradient and viscous term formulation given in [7,9]:

$$\left(-\frac{1}{\rho}\nabla p\right)_a = -\frac{1}{m_a}\sum_b (V_a^2+V_b^2)\tilde{p}_{ab}\nabla_a W_{ab} \tag{6}$$

$$\left(\frac{\mu}{\rho}\nabla^2\vec{v}\right)_a = \frac{1}{m_a}\sum_b\left(V_a^2+V_b^2\right)\tilde{\mu}_{ab}\frac{\vec{r}_{ab}\cdot\nabla_a W_{ab}}{\left|\vec{r}_{ab}\right|^2+\epsilon h^2}\left(\vec{v}_a-\vec{v}_b\right) \tag{7}$$