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Interaction solutions for a dimensionally reduced Hirota bilinear equation

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ABSTRACT

With the help of Hirota direct method and symbolic computation, two types of interaction solutions to the dimensionally reduced equations in (2+1)-dimensions are derived, respectively, by searching for the solutions to the associated bilinear equations as a combination of positive quadratic function and an exponential/hyperbolic cosine function. It is interesting that the interaction solutions between lump soliton and one stripe soliton generate the fission and fusion phenomena, while the interaction solutions between lump soliton and twin-stripe solitons generate the rogue wave.

1. Introduction

The exact solutions of nonlinear evolution equations (NLEEs) play an important role in understanding the nonlinear phenomena in nonlinear science [1]. Over the past decades, many effective methods have been developed to find the exact solutions of NLEEs, such as the Lie group method [2,3] and Hirota bilinear method [4]. With the help of Lie group method, Lie point symmetries of a differential system can be obtained, from which one can transform given solutions to new ones via finite transformation and construct group invariant solutions by similarity reductions. Recently, the nonlocal symmetry method is used to find exact interaction solutions among different nonlinear excitations, such as soliton–cnoidal wave solutions, soliton–Painlevé wave solutions, etc [5–10]. Inspired from the results obtained via nonlocal symmetry method, the consistent Riccati expansion method is further developed to construct the interaction solutions for Painlevé integrable system in a more direct but much simpler way [11–20].

Compared with the Lie group method, the Hirota bilinear method can be directly used to obtain the soliton solutions for NLEEs once its corresponding bilinear form is given. Based on the Hirota bilinear method, some simplified Hirota bilinear methods are developed to find various of exact solutions of NLEEs, such as the three-soliton method [21]. In Ref. [22], with the help of Hirota bilinear method, by taking function f in bilinear equation as a second order polynomial of x, y, z and t, the rational solutions and rogue wave solutions of a (3+1)-dimensional NLEE are obtained. In Ref. [23], a class of lump solutions which are rationally localized in all directions in the space of the KPI equation are presented by assuming the f as a positive quadratic function. It is interesting that by taking the f as a combination of positive quadratic function and exponential functions/hyperbolic functions, the interaction solutions such as lump–stripe solutions, lump–kink solutions, and even the rogue wave-typed solutions are successfully obtained [24–33].

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In this paper, we focus on the following Hirota bilinear equation [34,35]

$$(D_tD_y - D_x^3D_y - 3D_x^2 + 3D_z^2)f \cdot f = 0, \tag{1.1}$$

where $f = f(x, y, z, t)$ and the bilinear operators defined by [4]

$$D_{x_1}^{n_1} \cdots D_{x_l}^{n_l} F \cdot G = (\partial_{x_1} - \partial_{x_1'})^{n_1} \cdots (\partial_{x_l} - \partial_{x_l'})^{n_l} F(x_1, \ldots, x_l) \times G(x_1', \ldots, x_l')|_{x_1'=x_1,\ldots,x_l'=x_l}. \tag{1.2}$$

By means of the transformation

$$u = 2(\ln f)_x, \tag{1.3}$$

Eq. (1.1) can be rewritten as

$$u_{yt} - u_{xxxy} - 3(u_xu_y)_x - 3u_{xx} + 3u_{zz} = 0, \tag{1.4}$$

which is a (3+1)-dimensional NLEEs. Under the transformations [36]

$$x \to X, y \to X, z \to X, t \to -T, u_x \to U, \tag{1.5}$$

Eq. (1.4) reduces to the classical Korteweg–de Vries (KdV) equation

$$U_T + 6UU_X + U_{XXX} = 0, \tag{1.6}$$

which is a mathematical model of waves on shallow water surfaces. In Ref. [34], two types of resonant multiple wave solutions of Eq. (1.4) are obtained by applying the linear superposition principle. Ref. [36] obtained the rational breather wave and rogue wave solution of Eq. (1.4) by using homoclinic test method. Bäcklund transformation, multiple wave solutions and lump solutions of Eq. (1.4) are obtained in Refs. [35,37]. Lump solution and integrability [38] of the associated equation (1.4) are also studied by means of Hirota bilinear method and Bell polynomials.

The aim of this paper is to find the interaction solutions of the dimensionally reduced Hirota bilinear equation

$$(D_tD_y - D_x^3D_y - 3D_x^2 + 3D_y^2)f \cdot f = 0, \tag{1.7}$$

namely by taking $z = y$ in Eq. (1.1), which corresponding reduced (2+1)-dimensional equation reads

$$u_{yt} - u_{xxxy} - 3(u_xu_y)_x - 3u_{xx} + 3u_{yy} = 0. \tag{1.8}$$

In Section 2, a class of interaction solutions between lump soliton and one stripe soliton are derived by assuming function f in Eq. (1.7) as a combination of positive quadratic function and an exponential function. In Section 3, the rogue wave phenomenon is discovered during the process of interaction between lump soliton and twin-stripe-solitons, which this kind of interaction solutions are obtained by taking function f in Eq. (1.7) as a combination of positive quadratic function and a hyperbolic cosine function. Moreover, we also find no rogue wave during the process of interaction by selecting suitable solution for Eq. (1.7). Section 4 will be some conclusions.

2. Interaction solutions between lump soliton and one stripe soliton

The Hirota bilinear method has been shown to be used to find various exact solutions, such as resonant multiple wave solutions [39] and lump solutions [40]. In order to find the interaction solutions between lump soliton and one stripe soliton for Eq. (1.8), we firstly taking the function f in Eq. (1.7) as

$$f = g^2 + h^2 + l + a_9, \tag{2.1}$$

with

$$g = a_1x + a_2y + a_3t + a_4, h = a_5x + a_6y + a_7t + a_8, l = k\exp(k_1x + k_2y + k_3t), \tag{2.2}$$

where a_i $(1 \le i \le 9)$ and k, k_j $(1 \le j \le 3)$ are real parameters to be determined later. Symbolic computation on a direct substitution of Eq. (2.1) with (2.2) into Eq. (1.7) generates the following results

$$\begin{aligned} a_1 &= \frac{-4a_2a_6^2}{(a_2^2+a_6^2)k_2^2}, a_3 = \frac{-3a_2(a_2^6k_2^4 + 3a_2^4a_6^2k_2^4 + 3a_2^2a_6^4k_2^4 + a_6^6k_2^4 - (6a_2^2a_6^4 + 48a_6^6))}{(a_2^2+a_6^2)^3k_2^4}, a_5 = \frac{-4a_6}{(a_2^2+a_6^2)k_2^2},\\ a_7 &= \frac{-3a_6(a_2^6k_2^4 + 3a_2^4a_6^2k_2^4 + 3a_2^2a_6^4k_2^4 + a_6^6k_2^4 + 48a_2^2a_6^4 - 16a_6^6)}{(a_2^2+a_6^2)k_2^4}, a_9 = \frac{(a_2^2+a_6^2)(a_2^2-a_6^2)}{k_2^2a_2^2},\\ k_1 &= \frac{-4a_6^2}{(a_2^2+a_6^2)k_2}, k_3 = \frac{-(3a_2^6k_2^4 + 9a_2^4a_6^2k_2^4 + 9a_2^2a_6^4k_2^4 + 3a_6^6k_2^4 - 48a_2^2a_6^4 + 16a_6^6)}{(a_2^2+a_6^2)^3k_2^3}, a_4 = a_8 = 0, \end{aligned} \tag{2.3}$$