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Dynamic stability evaluation of underground tunnels based on deformation reinforcement theory

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ABSTRACT

Owing to the rapid and increasing application of underground engineering, the stability of underground tunnels under complex geological and seismic environments is receiving increasing attention. Seismic stability analysis and reinforcement design are important to guarantee engineering safety and operation. Based on the deformation reinforcement theory, a dynamic analysis method of the anti-seismic stability evaluation of underground tunnels is presented. The time history of plastic complementary energy norm is used to identify the instant when the structure is exhibiting the most devastating failure. The unbalanced force is used to represent the local failure of the structure. The reinforcement forces are derived through the distribution of unbalanced forces at the instant with the maximum PCE norm. These evaluation indices are incorporated into a three-dimensional nonlinear finite element analysis program, *TFINE*, which is designed by an object-oriented approach. Further, an elastoplastic integration algorithm and a comprehensive convergence rule are implemented. Examples of a single tunnel and parallel tunnels subject to earthquake are demonstrated, showing the capability and effectiveness of *TFINE* in evaluating the structure stability.

1. Introduction

A large number of underground caverns or tunnels have been deployed in many hydropower and traffic engineering [1]. Owing to the limitation of geological conditions, many hydropower stations built or being built in China employ the form of underground powerhouse, such as the Jinping-I hydropower station and the Xiluodu hydropower station. Meanwhile, in the transportation engineering constructions in Southeastern China, the tunnel tends to be the most economical structure form in terms of transport efficiency and transport cost, in that the tunnel would minimize the transporting distance and duration.

However, a majority of these rock engineering are located in complex geological environments that pose severe earthquake risks. Therefore, devastating geological hazards are likely to occur, and cause severe threats to the stability of the structure. The four cascaded hydropower stations in the lower reaches of the Jinsha River are situated in high and steep canyon regions, and are subject to intense earthquakes. For example, the basic earthquake intensity near the underground powerhouse of the operating Xiluodu hydropower station is approximately VIII [2]. The Xianglushan tunnel, which is the key water diversion project in the central Yunnan Province of China, involves the most complex geological problems, and the entire line is deeply buried. It passes through three large highly active fault zones and is subject to severe earthquakes and serious threats of gushing water [3]. Therefore, the dynamic stability issue in these engineering projects has raised deep concerns.

Many methods are traditionally used in engineering design and construction to determine whether the structure is stable and safe, e.g., convergence measurement or by the presence of a plastic yield area. In engineering practice, especially in tunnel construction, the observation of the tunnel convergence deformation is mandatory [5,6]. However, some difficulties and delay may occur in obtaining timely measurements [4]. Although the convergence deformation measurement can reflect some stress-induced problems and influences [7], the exact position where local failures occur cannot be identified through the displacement change because larger displacements do not guarantee larger strains or larger stresses. Meanwhile, in dynamic cases, the structure is subject to dynamic external loads, causing the displacement responses to change dynamically with time. Therefore, the stability of the structure is difficult to interpret simply through the variation in displacement of certain points.

The plastic region of the tunnel, which is often used to represent the yielding of the material, is an important reference to evaluate the stability of the tunnel. Zhu et al. [8] studied the distribution law of the plastic region to analyze the impact of in-situ stresses on the stability of the tunnel. Luan et al. [9] exploited the occurrence and evolution of the

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plastic region to predict the failure of the slope. However, the plastic region is only a qualitative description indicator representing that the material in the corresponding area has yielded or is beyond the yielding surface under external loads. It cannot quantitatively represent the extent of failure, i.e., describe whether the stress has only reached the yield surface or has extended far beyond the yield surface.

Arguably, the displacement convergence and the plastic region can only partially represent the stability state of the structure. Consequently, it is more important to look for other variables that can quantitatively represent the state of the global stability and the local failure of the structure under dynamic loads, such that the engineering design and repair scheme can be better guided.

With the development of computer techniques, the dynamic analysis of large and complex structures can be performed using numerical methods. Among numerous methods, the finite element method (FEM) has received a wider application and many excellent commercial FEM software have emerged.

Hence, a high-efficiency method of dynamic stability analysis is presented and is applied to underground rock engineering. Based on the deformation reinforcement theory (DRT, [15]), the instantaneous plastic complementary energy (PCE) norm is derived from the concept of overstress, and the time history of the PCE norm is utilized to quantify the global stability of the structure under dynamic loads; further, the unbalanced forces are used to represent the extent of the local failure. The proposed analysis method is implemented in the three-dimensional nonlinear dynamic finite element analysis program, *TFINE*. A return mapping plasticity algorithm based on the Drucker–Prager yield criterion and an analytic solution to the transferred stresses are derived [18,19] to enhance the convergence and efficiency of the nonlinear elastoplastic computing.

Herein, the dynamic extension of DRT is presented in Section 2, where the derivations of the PCE norm and unbalanced forces are reviewed and their usage in the stability analysis of structures is illustrated. In Section 3, the dynamic analysis is briefly reviewed. In Section 4, the frame of the dynamic finite element program, *TFINE*, is demonstrated; additionally, the constitutive integration algorithm and a comprehensive convergence rule are elaborated with their implementation in the program presented. Subsequently, examples of a single tunnel and parallel tunnels are analyzed using *TFINE* to demonstrate the effectiveness of the proposed dynamic stability analysis method in underground engineering.

2. Dynamic deformation reinforcement theory

The stability of the structure is of deep concern in engineering design and operation. The classic elastoplastic theory requires all the stresses of a stable structure to be inside or on the yielding surface. After sufficient iteration, the stresses beyond the yield surface would be adjusted and returned to the yield surface, and the unbalanced forces would be iteratively eliminated to zero or adequately small such that the external loads are in equilibrium with the internal resisting forces. Otherwise, if the iteration is not converged eventually, the structure is considered to be unstable or in failure. In other words, traditional finite element analysis cannot yield useful information after the structure has experienced the non-equilibrium state when the external loads exceed the resistance capability of the structure, and would not know the difference in the failure impact of different loads on the same structure.

Yang et al. [17–20] proposed the DRT that is based on non-equilibrium elastoplastic mechanics. In this theory, non-equilibrium plastic regions are focused more. A concept of the PCE norm, i.e., a norm representation of plastic stress, is presented to quantitatively describe the failure extent of the structure. Further, the unbalanced forces, i.e., the equivalent nodal forces of the plastic stresses, are regarded as the minimum reinforcement forces required to maintain the stability and balance of the structure.

Meanwhile, the DRT has been primarily used for static cases of various types of geotechnical engineering. This section presents a dynamics extension of the DRT such that it is applicable in the stability analysis of dynamic cases.

2.1. *Instantaneous plastic complementary energy norm*

The displacement and stress fields of the global structure should simultaneously satisfy the equilibrium condition, kinematical admissibility, and constitutive equations. This principle is also used implicitly to obtain the point of impending instability [15,27–29].

The DRT is based on incremental elastoplastic constitutive equations [17–20]. The PCE norm is an inner product of the difference of two stress fields in the Euclidean space, where one is a compatible equilibrium stress field and the other is a compatible stable stress field. To maintain the structure stability in dynamic cases, the equilibrium condition and the yield criterion must be satisfied at all points; therefore, the DRT holds in dynamic cases as well.

The most general solution method for dynamic analysis is an incremental method in which the equilibrium equations are solved at times Δt, $2\Delta t$, $3\Delta t$, etc. A large number of different incremental solution methods exist. In general, they involve a solution of the complete set of equilibrium equations at each time increment [10,12].

In addition to the dynamic external loads, the structure is subject to effects of inertia force and damping force. If a displacement-based solution is chosen (Newmark, [11]), these items can be transformed into equivalent external loads, and the stiffness of the structure is transformed into the equivalent stiffness. The starting stress field of the structure is $\boldsymbol{\sigma}_t^0$ at the beginning of every time step, and the equivalent external loads produce a displacement increment Δu_t and accordingly a strain increment $\Delta\varepsilon_t$. The strain increment produces an elastic loading stress state $\boldsymbol{\sigma}_t^1 = \boldsymbol{\sigma}_t^0 + \Delta\boldsymbol{\sigma}_t^e$ when $f(\boldsymbol{\sigma}_t^1) \le 0$, where $\Delta\boldsymbol{\sigma}_t^e = \boldsymbol{D}\colon \Delta\boldsymbol{\varepsilon}_t$. Plastic loading is produced by the strain increment if $f(\boldsymbol{\sigma}_t^1) > 0$; subsequently, the final stress state has to be adjusted to $\boldsymbol{\sigma}_t = \boldsymbol{\sigma}_t^1 - \Delta\boldsymbol{\sigma}_t^p$, where $f(\boldsymbol{\sigma}_t) = 0$.

By analogy with the PCE norm definition ΔE in the static case [18], and taking the compliance tensor $\boldsymbol{C}/2$ as the metric tensor, the instantaneous PCE norm at time t can be defined as

$$\Delta E_t = \frac{1}{2}\int_V (\boldsymbol{\sigma}_t^1 - \boldsymbol{\sigma}_t)\colon C\colon (\boldsymbol{\sigma}_t^1 - \boldsymbol{\sigma}_t)dV$$

When the structure is subject to dynamic external loads, the equilibrium stress field set $\boldsymbol{S}_1$ will change dynamically with time, e.g., $\boldsymbol{S}_1^{t_1}$ at $t = t_1$ and $\boldsymbol{S}_1^{t_2}$ at $t = t_2$. But the stability stress field is constant as time varies if infinitesimally small deformation and small strain are assumed, and the damage-induced weakening of the materials is not considered. When a solution that simultaneously satisfies the equilibrium and the stability stress field does not exist, the structure enters the state of non-equilibrium or fails. As analyzed in the DRT, the elastoplastic iterations within each time step finally result in a minimum PCE norm, which represents the instability extent of the structure under the instantaneous external loads at that moment. $\Delta E(t_1)$ and $\Delta E(t_2)$ are obtained at $t = t_1$ and $t = t_2$, respectively. Therefore, a $\Delta E_t \sim t$ curve can be drawn as the PCE norm varies with time, e.g., Fig. 2 that shows an example of the PCE norm time history of a rock structures surrounding a deep buried tunnel subject to certain earthquake loads.

Because the PCE norm quantitatively represents the global distance from the equilibrium state under the current loads to the stable state of the structure, it can be considered that, for time of t' when $E_{t'} = \max(E_t)$, the equilibrium stress field is farthest from the stable stress field, i.e., the structure is experiencing the most devastating external loads, and accordingly is most likely to fail at that moment. Additionally, because the PCE norm derivative $\frac{d\Delta E_t}{dt}$ indicates the varying rate of the global stability, the larger the PCE norm rate, which means the faster the structure of unstable state moves to a more unstable state, for brittle materials, the more likely is the structure to fail.

In summary, the PCE norm time history is highly useful in dynamic