Mechanics of Materials 127 (2018) 14–25

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Research paper

Closed-form expressions for effective viscoelastic properties of fiber-reinforced composites considering fractional matrix behavior

U. Hofer*, M. Luger, R. Traxl, R. Lackner

Material Technology Innsbruck (MTI), University of Innsbruck, Technikerstraße 13, Innsbruck A-6020, Austria

ARTICLE INFO

Keywords:
Fractional viscoelasticity
Fiber-reinforced-polymers
Effective properties

ABSTRACT

For determination of effective elastic properties of fiber-reinforced materials the so-called Chamis-equations are standardly used (Chamis, 1989). In this paper, these equations are extended towards viscoelasticity, making use of the correspondence principle (Lee, 1955). The determination of the effective properties takes place in the Laplace–Carson domain, with the back-transformation to the time domain being performed analytically, providing closed-form expressions for the effective viscoelastic behavior of the composite. Fractional viscoelastic models are used to describe the time-dependent behavior of the matrix, well-capturing the experimentally-observed viscoelastic behavior with a comparatively low number of parameters. The performance of the proposed determination of effective viscoelastic properties is assessed by means of alternative methods such as Eigenstrain-based methods and numerical simulations employing unit-cell (UC) models.

1. Introduction

Fractional viscoelastic models (see Fig. 1) are standardly employed to describe creep and relaxation of a variety of materials (Podlubny, 1999; Mainardi, 2010): Yancey and Pindera (1990) used a fractional Maxwell model to represent creep data of epoxy resin; in (Dinzart and Lipiński, 2010) a fractional Zener model is used to model creep of PMMA (polymethyl methacrylate) and SAN (styrene acrylonytrile copolymer); Pichler et al. (2012) identified a fractional Maxwell model for the viscoelastic behavior of bitumen, with Oeser et al. (2008) and Celauro et al. (2012) using fractional Burgers models to fit creep data of asphalt mixtures; moreover, soft biological tissues are found to obey a fractional Kelvin-Voigt model as proposed in (Meral et al., 2010). Representing the power-law model, i.e. the fractional Maxwell model, by a mechanical spring-dashpot combination, Deseri et al. (2014) found a unique expression for the corresponding free energy, making the fractional Maxwell model even more attractive for being used in viscoelastic material modeling.

For materials consisting of more than one material phase, its effective (homogenized) elastic as well as viscoelastic behavior may be determined from the behavior of its constituents by means of homogenization methods: Hashin and Rosen (1964) derived expressions for the effective moduli of fiber-reinforced materials by using the composite cylinder assemblage model, consisting of cylindrical fibers embedded in a matrix. The elastic strain energy is formulated for a kinematically admissible strain field and a statically admissible stress field, leading to lower and upper bounds for the strain energy. Expressions for the effective elastic moduli are finally found by comparing the strain energy of the cylinder assemblage model with the strain energy of the effective material. The same author (Hashin, 1966) extended the aforementioned method to viscoelastic behavior by applying the correspondence principle (Lee, 1955), where the viscoelastic matrix material behavior was represented by a Maxwell model. The back-transformation to time domain was performed analytically assuming rigid fiber behavior. In (Upadhyaya and Upadhyay, 1991), Hashin's composite cylinder model was employed to obtain the effective viscoelastic properties of unidirectionally-reinforced composites, with the viscoelastic material behavior of the matrix extended towards a Prony series, i.e. a generalized Maxwell model.

Using the equivalent Eigenstrain method (Nemat-Nasser and Hori, 1993), the effective elastic properties of a medium with inclusions are determined. An Eigenstrain field is applied onto an effective medium in such a way, that it represents the strain field of the respective heterogenous material. By comparing the stress and strain fields, respectively, in the heterogenous and homogenous medium, the elastic stiffness tensor is obtained. For ellipsoidal inclusions, the solution given in (Eshelby, 1957) leads to explicit expressions for the elastic properties. For cylindrical inclusions instead, i.e. for fiber-reinforced composites, explicit expressions for the elastic properties were presented in (Nemat-Nasser and Hori, 1993), which have been adapted

* Corresponding author.
E-mail address: Ulrich.Hofer@uibk.ac.at (U. Hofer).

https://doi.org/10.1016/j.mechmat.2018.08.005
Received 5 February 2018; Received in revised form 24 July 2018; Accepted 16 August 2018
Available online 18 August 2018
0167-6636/

(Luciano and Barbero, 1995; Barbero and Luciano, 1995) in order to obtain analytical expressions for the effective relaxation moduli of unidirectionally-reinforced composites considering a Burgers model for the viscoelastic behavior of the matrix material. By application of the correspondence principle, the viscoelastic problem is solved in the Laplace–Carson domain analogously to the elastic problem.

In an adaption of the equivalent Eigenstrain method, i.e. the self-consistent scheme, the effective medium is used for the medium surrounding the inclusions instead of the pure matrix (Budiansky, 1965; Hill, 1965). This scheme was extended towards viscoelasticity in (Laws and McLaughlin, 1978) for spheroidal inclusions and cylindrical fibers by applying the correspondence principle. Finally, Dinzart and Lipiński (2010) extended the self-consistent method towards fractional viscoelastic behavior, representing the polymeric constituents by a fractional Zener model. Experimental validation was exclusively performed in the Laplace–Carson domain employing cyclic-test results; hence, back-transformation of the effective relaxation moduli to the time domain became obsolete.

Wang and Pindera (2016) extended an elastic homogenization theory introduced by Drago and Pindera (2008) towards viscoelasticity by using the correspondence principle. The viscoelastic matrix behavior was represented by a fractional Maxwell model, the effective viscoelastic properties were back-transformed to the time domain numerically.

In this paper, a closed-form solution for the effective viscoelastic behavior of fiber-reinforced composites considering fractional viscoelastic matrix behavior is presented. For this purpose, the Chamis-equations – standardly employed for determination of the effective elastic properties – are extended towards viscoelasticity. The homogenization is performed in the Laplace–Carson domain. By means of back-transformation to the time domain, closed-form expressions for the effective viscoelastic moduli are found.

This paper is organized as follows: In Section 2, fractional viscoelasticity is briefly reviewed, dealing with the fractional Burgers model, covering the fractional Zener, Maxwell, and Kelvin-Voigt models as special cases. The extension of the Chamis-equations towards viscoelasticity is treated in Section 3, finally providing closed-form expressions for the effective viscoelastic properties. In Section 4, the performance of the extended Chamis-equations is assessed by alternative homogenization methods for determining the effective viscoelastic behavior, such as the equivalent Eigenstrain method and numerical UC-based simulations. The paper closes with concluding remarks in Section 5.

2. Fractional viscoelasticity

For many materials, the viscoelastic behavior is mainly driven by the deviatoric part of the stress state (Lakes and Wineman, 2006). Therefore, constitutive equations are split into bulk and shear behavior, linking volumetric and deviatoric stresses to the respective strains, reading in case of isotropic elastic behavior:

$$\boldsymbol{\sigma} = \mathbb{C}:\boldsymbol{\varepsilon} \quad \Leftrightarrow \quad p = K\varepsilon^{vol}, \quad \boldsymbol{s} = 2\,G\boldsymbol{e}, \tag{1}$$

with K and G as bulk and shear modulus, respectively. In Eq. (1), $p = \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})$, $\varepsilon^{vol} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, and $\boldsymbol{s}$ and $\boldsymbol{e}$ represent the stress and strain deviator, respectively. In case of viscoelastic deviatoric behavior, the respective stress history is expressed by the convolution integral, reading (Christensen, 1982)

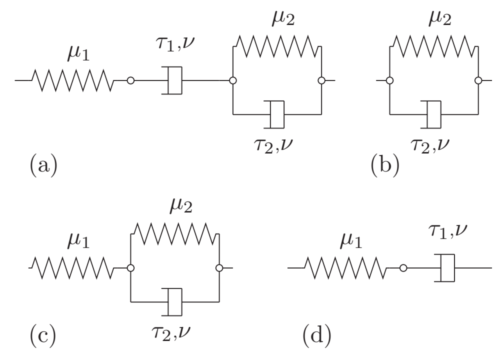

Fig. 1. Mechanical representations of fractional models: (a) fractional Burgers model, (b) fractional Kelvin-Voigt model, (c) fractional Zener model, and (d) fractional Maxwell model.

$$\boldsymbol{s}(t) = 2\int_0^t R(t-t')\frac{d\boldsymbol{e}(t')}{dt'}dt' \tag{2}$$

assuming $\boldsymbol{s}(t) = \boldsymbol{0}$ for $t < 0$. In Eq. (2), $R(t)$ refers to the relaxation modulus, with $R = G$ in case of elastic behavior. The deviatoric relaxation modulus $R(t)$ depends on the underlying viscoelastic model, reading for the fractional Burgers model considered in this paper (Mainardi and Spada, 2011):

$$R(t) = 2\mu_1\sum_{i=1}^{2} R_i E_\nu\left[-\left(\frac{t}{\tau_0\rho_i}\right)^\nu\right] \tag{3}$$

where the following abbreviations are used:

$$\begin{aligned}
r_\mu &= \frac{\mu_1}{\mu_2}, \quad r_\tau = \frac{\tau_1}{\tau_2}, \quad \tau_0^\nu = \frac{(\tau_1\tau_2)^\nu}{\tau_1^\nu + \tau_2^\nu}\\
\delta_0 &= \frac{1}{1+r_\mu}\frac{r_\tau^\nu}{(1+r_\tau^\nu)^2}, \quad \delta_1 = \frac{1}{1+r_\mu} + \frac{r_\mu}{1+r_\mu}\frac{r_\tau^\nu}{1+r_\tau^\nu},\\
&\delta_2 = \frac{1}{1+r_\mu}, \quad \gamma_0 = \frac{r_\tau}{1+r_\tau}\\
z_{1,2} &= -\frac{\delta_1}{2\delta_2} \pm \sqrt{\frac{\delta_1^2}{4\delta_2^2} - \frac{\delta_0}{\delta_2}}, \quad \rho_1^\nu = -\frac{1}{z_1}, \quad \rho_2^\nu = -\frac{1}{z_2},\\
&R_1 = \frac{z_1+\gamma_0}{z_1-z_2}, \quad R_2 = \frac{z_2+\gamma_0}{z_2-z_1},
\end{aligned} \tag{4}$$

with μ_1, μ_2 [Pa], τ_1, τ_2 [s], and ν [-] referring to the parameters of the springs and fractional dashpots as illustrated in Fig. 1. The Mittag-Leffler function employed in Eq. (3)is defined as (Mainardi and Spada, 2011):

$$E_\alpha(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}, \alpha > 0, z \in \mathbb{C} \tag{5}$$

Considering $2\mu_1$ being equal to the elastic shear modulus G, the dimensionless relaxation modulus $r(t)$ is introduced as

$$r(t) = \frac{R(t)}{G} = \sum_{i=1}^{2} R_i E_\nu\left[-\left(\frac{t}{\tau_0\rho_i}\right)^\nu\right] \tag{6}$$

3. Viscoelastic extension of Chamis-equations

The Chamis-equations provide the effective elastic properties (Young's moduli, shear moduli, Poisson's ratios) for a unidirectionally-reinforced composite (Chamis, 1989):