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Impact of Radon transform on plane wave diffraction by an impedance step geometry

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ABSTRACT

This work presents a Radon transform analysis of plane wave diffraction by an impedance step, such that the planes and the step have different impedances. Application of Radon transform on diffraction analysis is of vital importance in many real-time applications *i.e.,* medical imaging, geosciences, astronomy, optics, stress analysis *etc.* Radon transform offers the advantage of introducing freedom in the usage of observer angle, during reconstruction of signal using plane wave diffraction. Radon transform along with Wiener–Hopf technique and asymptotic method of integration are used for computing the far field of the diffracted plane wave. The solution of plane wave diffraction by an impedance step consists of an infinite set of linear equations, which is obtained with the help of a software. Sinograms showing the effect of varying step height and impedance on the scattered field are plotted, compared and discussed. The mathematical analysis and graphical discussion show that Radon transform enables the observer angle to be varied between 0 to 2π. The results are comparable to previous works on plane wave diffraction by an impedance step, when the observer angle is fixed.

1. Introduction

Diffraction of signal from a step geometry is an important phenomenon that has mathematical as well as physical implications. Step geometry is used to interconnect circuits in many electrical and electronic devices e.g., solder-pad, microwave ovens and frequency selective surfaces [1]. Johansen appears to be the first to study diffraction by a step connecting two semi-infinite planes, where the steps and the plane have same impedances [2]. Johansen's work has been further extended by considering different impedances for semi-infinite planes and the connecting step in [3,4]. Recently, diffraction from step geometry has also been investigated by in [5–7].

Radon transform has been widely used in the field of science and engineering. Physically, Radon transform produces $n-1$ dimensional plane of an n dimensional function. In a three dimensional space, the volume is detected using the line integral of one point in a projection [8]. Some applications of Radon transform include reconstruction tomography (RT) [9] *i.e.,* computed tomography (CT), X-ray CT, Emission CT, ultrasound CT, astronomy, electron microscopy, nuclear magnetic resonance (NMR), optics, stress analysis and geophysics [9] *etc.*

The step geometry is important in medical imaging since, many bones, tissues and joints can be approximated by a step geometry. Earlier studies on plane wave diffraction using impedance step geometry are limited in their ability to change in observer angle and

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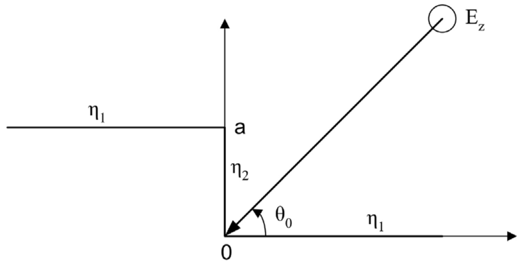

Fig. 1. Geometry of diffraction problem.

assume the observer angle to be fixed. The observer angle must be varied for reconstructing a signal whose most common application can be observed in medical imaging. The objective of this work is to compute and construct the sinograms of the diffracted fields patterns, which are generated by diffraction of plane waves from an impedance step using Radon transform analysis. The expression computed for diffraction from a step geometry using Radon transform contains observer angle as a variable and thus offers the flexibility to change the observer angle between 0 to 2π. To the best of our knowledge, no evidence is available in existing literature that studies the effect of Radon transform analysis on the sinograms of the diffracted field produced by a step geometry.

In this paper, a Radon transform analysis of the diffracted field of an impedance step and plane having different impedances is performed. The mathematical analysis and graphical discussion show that utilizing Radon transform offers freedom in varying observer angle. The sinograms and the final expression of the solution are comparable to previous works, when the observer angle is fixed.

The remainder of the paper is organized as follows. Section 2 formulates the problem and describes the mathematical solution. Section 3 describes the radon transform of the diffracted field obtained in Section 2. Section 4 gives the graphical illustrations and their discussion. Finally conclusions are presented in Section 5.

2. Mathematical analysis of the problem

Fig. 1 shows the geometry of problem under consideration. A plane wave illuminating two half planes (having impedance η_1) joined by a step (having impedance η_2) is of the form

$$\chi^i(x, y) \equiv E^i(x, y) = \exp - \mathrm{ik}(x\cos\varphi_0 + y\sin\varphi_0), \tag{1}$$

with k being the wave number having small positive part and φ_0 is the angle of incidence. A time dependence of the $\exp(-i\omega t)$ is assumed and dropped in the foregoing analysis. The total velocity potential χ_t satisfying the wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)\chi_t(x, y) = 0, \tag{2}$$

subject to the boundary condition on a half plane is

$$\chi_t(x, y) + \frac{\eta_1}{\mathrm{ik}}\frac{\partial\chi_t(x, y)}{\partial y} = 0, \quad \text{at} - \infty < x < 0, y = a, \tag{3}$$

on the step is given by

$$\chi_t(x, y) + \frac{\eta_2}{\mathrm{ik}}\frac{\partial\chi_t(x, y)}{\partial x} = 0, \quad \text{at} x = 0, 0 < y < a, \tag{4}$$

and the boundary condition on the other step is

$$\chi_t(x, y) + \frac{\eta_1}{\mathrm{ik}}\frac{\partial\chi_t(x, y)}{\partial y} = 0, \quad \text{at} 0 < x < \infty , y = 0. \tag{5}$$

In addition to the boundary conditions (3)–(5), the following continuity conditions at $0 < x < \infty, y = a$ should also be satisfied to compute the solution

$$\chi_t(x, a^+) = \chi_t(x, a^-), \tag{6}$$

$$\frac{\partial\chi_t(x, a^+)}{\partial y} = \frac{\partial\chi_t(x, a^-)}{\partial y}, \tag{7}$$

The superscripts “ + ” and “ − ” indicate that the step is approached either from left or from right hand side along the x-axis. For the purpose of analysis, it is convenient to decompose the field $\chi_t(x, y)$ as follows