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# Valuation of n-fold compound barrier options with stochastic interest rates

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This paper considers the pricing of n-fold compound options with barriers. It can apply to financial derivatives with credit risk and real option applications with the right to abandon prior to maturity based on compound option theory. According to the correlation between barriers in intervals, there are two cases. One is the case of independent barriers, since the underlying value has uncorrelated default boundaries during different time intervals, while the other has correlation and results in a loosened default barrier stage by stage. Additionally, we develop a generalization of compound barrier options with stochastic interest rates to capture the interest rate risk. Finally, the characteristics of the model are illustrated with numerical examples. We find the following three results. First, the down-and-out barrier brings an early termination premium if the option is likely to be out-of-the-money in the future. Second, the loosened barrier in the case of dependent barriers has less probability of being knocked out as soon as passing through an earlier passage time. Third, a compound option with a barrier is more difficult to hedge, while increasing the number of folds reduces this difficulty.



## 1. Introduction

Compound options are simply options on options. That is, their underlying assets are other options. Treating corporate liabilities as options was first proposed in Black and Scholes (1973), while Merton (1974) applies these ideas to price corporate debt. Geske (1977) derives a formula to value coupon bonds and subordinated debt as compound options. The analytical formula for the price of compound options is worked out in Geske (1979). The compound option theory has been extensively used in finance derivatives. Various corporate securities, such as equities, warrants, and convertible bonds, can be valued as claims contingent on a firm's assets. Since the structure of financial derivatives has become increasingly complex, the formula for 2-fold compound options proposed by Geske (1979) may not be capable of analyzing these more sophisticated financial instruments. Moreover, the 2-fold compound options pricing formula cannot be used as a further building block to construct more sophisticated approaches. Specific n-fold compound options pricing formulas were thus suggested by Geske and Johnson (1984a, 1984b) and Carr (1988), while formulae for pricing sequential compound calls were also provided by Thomassen and Van Wouwe (2001, 2002), Chen (2003) and Lee, Yeh, and Chen (2008).

Credit risk is becoming an issue of greater importance, with global credit quality deteriorating and the volume of corporate bonds rising dramatically. One severe credit event that took place relatively recently was the US sub-prime mortgage crisis, which started in 2007. This led to billions in losses by banks, a slowdown in the US economy and a shock to the global financial market. Much of the recent finance literature has thus paid attention to default risk by developing numerous credit risk models, and these can be categorized into reduced form and structural models. Reduced form models use the market credit spread to obtain the probability of default and the mean recovery rate. While structural models use the capital structure variables of a firm, such as asset and debt values, to obtain the probability of default and the mean recovery rate. This provides a link between a firm's credit quality and its economic and financial conditions. Therefore, defaults occur endogenously within the model instead of exogenously, as in the reduced form models.

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The structural literature on credit risk begins with Merton (1974), in which the firm's default risk can be linked to standard option theory. If we treat a firm's assets as the underlying asset and the promised payment of corporate liabilities as the strike price, then the firm's equity can be thought of as a standard call option on the market value of the firm's asset. However, this approach has not been without critics. Black and Cox (1976) point out that one drawback of this method is that default only occurs at maturity of the debt. They suggest that a barrier on the market value of a firm's assets should be incorporated into the model for triggering default prior to the maturity. This is the so-called first passage time model. The down-and-out call option was thus proposed to model the firm's equity value, and the default risk can be estimated from the down-and-out call option pricing model. However, only corporate securities, but not options, are priced in Black and Cox (1976). Toft and Prucyk (1997) investigate the capital structure of perpetual debt and price stock options based on their firm values. Ericsson and Reneby (1998, 2003) propose a model to price the European compound options underlying corporate securities with the possibility of default. Therefore, in this paper, these ideas are built into n-fold compound options by using European barrier options as building blocks. The obtained formula can price the financial derivatives with credit risk prior to maturity.

Additionally, compound option theory is widely used in the study of real options. This approach originates from Myers (1977) and was developed by Brennan and Schwartz (1985), Pindyck (1988), Trigeorgis (1993, 1996) and so on. Some examples include the project valuation of new drugs suggested by Cassimon, Engelen, Thomassen, and Van Wouwe (2004), production and inventory by Cortazar and Schwartz (1993) and capital budget decisions by Duan, Lin, and Lee (2003). Moreover, some studies related to stochastic investments often use barriers as the investment triggers. Brock, Rothschild and Stiglitz (1989) note the possibility of extending a real options model with barriers. Saphores (2002) warns that the standard real options approach usually leads to incorrect investment decision rules, and proposes optimal investment rules with different barriers to revisit the simplest stochastic investment decision: when to incur a sunk cost in exchange for a random payoff. Ha-Duong and Morel (2003) investigates the theoretical problem of real options with absorbing barriers. An investment decision is modeled with a double irreversibility concern: investing is irreversible, but waiting runs the risk of loosing the opportunity to invest. The investment opportunity disappears if the project value ever reaches some specific investment barrier level. These results are especially important today, as more people use a real options approach to deal with decision-making problems under uncertainty. For instance, competitors' changes in market demand can be modeled as an absorbing barrier or an investment trigger, other examples include customers' gradual changes in tastes, the investment opportunity represented by a call option to takeover another firm, the exhaustibility of a natural resource, and so on.

The n-fold compound barrier option can have a number of practical applications, not only in corporate finance, which incorporates financial distress, but also in the real options field with the use of absorbing barriers. According to the different settings of the first passage time model, we can analyze two cases. One is the case of independent barriers, since the underlying value has uncorrelated default boundaries during different time intervals, while the other has a correlation and results in a loosened default barrier stage by stage.

For financial derivatives, Geske (1977) gives a closed-form expression for the price of a defaultable coupon bond in some models that is an extension of Merton's approach. Geske considers the option that stockholders have at each coupon payment of buying the next option or not. A defaultable coupon bond with sequential payouts can thus be valued as a compound option. Our resulting formula contains multivariate normal distribution functions, where $n$, equal to the size of the highest dimensional distribution function, is the number of nested options in the sequence of payouts. Furthermore, hybrid financial instruments with sophisticated features are applied to our derived model. Ananthanarayanan and Schwartz (1980) indicate that bonds with extended maturities can be regarded as compound options. On the first maturity date, the bondholder has two choices. He can either extend the maturity of the bond or receive the face value. The relative magnitude of the face value and the present value of the extended bond on the first maturity date would influence this decision. Gong, He, and Zhu (2006) employ the concept of n-fold compound options to price convertible bonds which provide holders with a right to convert the bond into a predetermined number of stocks with a pre-specified price, or to hold the bond till maturity to receive coupons and the principal. Ingersoll (1977) and Brennan and Schwartz (1977) are the premier researchers on convertible bond pricing, and they both use the contingent claims approach to the valuation of convertible bonds. The price of the convertible bond depends on the firm value as the underlying variable in their models. A drawback of these papers is that they ignore the likelihood of financial distress before maturity in a firm. Our model considers the default barrier to enhance the application of the compound option approach.

Multi-stage investment projects are frequently modeled as compound options in real options applications. For instance, there are several stages in a mining investment, which includes extraction, milling, smelting and refinery. The output of the previous stage is the input of the current stage and there are additional costs before going to the next stage. Paddock, Siegel and Smith (1988) note that three stages, i.e. exploration, development and extraction, are needed for a firm to exploit petroleum. Alvarez and Stenbacka (2001) indicate that the exercise of a presently available real option opens the opportunity for the implementation of future real options. Cassimon et al. (2004) also propose an investment cycle project in the pharmaceutical sector, where R&D is followed by a pre-clinical test phase, several clinical test phases, and larger scale market entry only after a positive decision by some regulatory body. We know that the n-fold compound option approach has been applied to real options. Here, barriers can be thought of as the right to abandon a project early if its cash flows in the future are lower than the salvage value, such as the market value of equipment used in the project.

In particular, one valuation of an n-fold compound barrier option is an installment option. An installment option is an option in which the premium is usually spread out in equal amounts over time. At each installment date, the holder of the option makes a decision about whether to exercise it, thereby paying the installment premium and continuing with the option. If the holder prefers not to continue, he or she simply fails to pay the installment, and the option then terminates. The earliest article on a standard installment option seems to be Karsenty and Sikorav (1993), while more recently Davis, Schachermayer and Tompkins (2001, 2002) state that the case of a two-period installment payment option is equivalent to a compound option, as previously considered by Geske (1979). Griebsch, Kuhn and Wystup (2004, 2007) further derive a closed-form solution to the value of an n-period installment option. They indicate that such an option can be understood as a series of n options, and the first one depends on the next.

A traded installment option can be applied in international treasury management, and this has the following advantages. One is the prevention of losses through the possibility of termination, while another is that it can help in situations where the need to hedge is uncertain. A third benefit is that the low initial premium can be easy to schedule in the firm's budget. As such, firms or investors can buy an installment call option to hedge the costs if they

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