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Sparse inference of the drift of a high-dimensional Ornstein–Uhlenbeck process

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Abstract

Given the observation of a high-dimensional Ornstein–Uhlenbeck (OU) process in continuous time, we are interested in inference on the drift parameter under a row-sparsity assumption. Towards that aim, we consider the negative log-likelihood of the process, penalized by an ℓ^1-penalization (Lasso and Adaptive Lasso). We provide both finite- and large-sample results for this procedure, by means of a sharp oracle inequality, and a limit theorem in the long-time asymptotics, including asymptotic consistency for variable selection. As a by-product, we point out the fact that for the Ornstein–Uhlenbeck process, one does not need an assumption of restricted eigenvalue type in order to derive fast rates for the Lasso, while it is well-known to be mandatory for linear regression for instance. Numerical results illustrate the benefits of this penalized procedure compared to standard maximum likelihood approaches both on simulations and real-world financial data.

Keywords: High-dimensional statistics, Lasso, Ornstein–Uhlenbeck process, Sparse estimation
MSC 2010. 60G15; 62H12; 62M99

1. Introduction

The Ornstein–Uhlenbeck, also called mean-reverting diffusion process, describes a process which evolves following a deterministic linear part with an added Gaussian noise, similarly to a vector-autoregressive process in discrete time. This model is ubiquitous in quantitative finance; for instance, the one-dimensional version is used for modeling rates and is called the Vašíček model [18]. In a multi-dimensional setting, therefore, it can be used to describe systems with linear interactions perturbed by Gaussian noise; see Figure 1. Among many others, an example of application is inter-bank lending [9, 12], where lending is a flux of reserves and is proportional to the difference in reserves.

A natural question is therefore how to estimate the interaction structure from the observation of the process. Unfortunately, the optimal solution based on the maximum likelihood estimator (MLE) is typically quite inaccurate in high-dimensional settings, because of the well-known curse of dimensionality; see, e.g., [6]. Moreover, in real-world applications, the interaction structure is sparse: in the example mentioned above, banks have typically only a few lending partners [5, 13, 14]. The idea of the Carmona–Fouque model [9] is to say that the diagonal terms of the matrix parameter $\boldsymbol{A}_0$ correspond to the outflows of liquidity and the non-diagonal terms to the inflows of liquidity. Row sparsity means that each actor receives capital only from a limited number of institutions, which has a natural meaning for interbank lending. Note that asymmetry of loan relationships and default propagation situations is not under the scope of this work.

In this paper, we exploit the sparsity property by using a sparsity-inducing penalization. Sparse inference using convex penalization has developed quickly in the past decade [1, 6, 15], mostly for linear supervised learning. Prior to the present work, there is surprisingly only one previous attempt to use these techniques in the setting of diffusion processes, in particular for the Ornstein–Uhlenbeck diffusion considered here [26], with no theoretical guarantees nor applications on real-world data. The aim of this paper is therefore to fill this gap, and to give a complete theory for this case, by developing both non-asymptotic results by means of a sharp oracle inequality, see Section 2, and asymptotic

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