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On APF test for Poisson process with shift and scale parameters

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ABSTRACT

We propose the goodness of fit test for inhomogeneous Poisson processes with unknown scale and shift parameters. A test statistic of Cramér–von Mises type is proposed and its asymptotic behavior is studied. We show that under null hypothesis the limit distribution of this statistic does not depend on unknown parameters.

1. Introduction

The problems of the construction of goodness of fit tests in the case of i.i.d. observations are well studied (Lehmann and Romano, 2005). Special attention is payed to the case of parametric null hypothesis. Wide class of distributions can be parametrized by the shift and scale parameters, say, $F\left(\frac{x-\vartheta_1}{\vartheta_2}\right)$. In the case of such families several authors showed that the limit distributions of the Kolmogorov–Smirnov and Cramer–von Mises tests statistics do not depend on the unknown parameters (see Darling, 1958; Durbin, 1973; Gikhman, 1953; Dzhaparidze and Nikulin, 1982; Martynov, 1979 and references therein). We call such tests *asymptotically parameter free* (APF).

For the continuous time stochastic processes the goodness of fit testing is not yet well developed. We can mention here several works for diffusion and Poisson processes (Dabye, 2013; Dabye et al., 2016; Dachian and Kutoyants, 2007; Davies, 1977; Kleptsyna and Kutoyants, 2014; Kutoyants, 2014b, a; Weiss, 1975). The problem of goodness of fit testing for inhomogeneous Poisson process is interesting because there is a wide literature on the applications of inhomogeneous Poisson process models in different domains (astronomy, biology, image analysis, medicine, optical communication, physics,

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reliability theory, etc.). Therefore to know if the observed Poisson process corresponds to some parametric family of intensity functions is important.

We consider the problem of goodness of fit testing for inhomogeneous Poisson process which under the null hypothesis has the intensity function with shift and scale parameters (two-dimensional unknown parameter). We show that as in the classical case the limit distribution of the Cramer–von Mises type statistics does not depend on these unknown parameters. This allows us to construct the corresponding APF goodness of fit test of fixed asymptotic size. The similar one-dimensional problems with shift and scale parameters separately were considered in the works (Dabye, 2013; Dabye et al., 2016) respectively. The proofs there are based on the asymptotic expansions of the MLEs of these parameters obtained in Kutoyants (1998). These expansions require much more regularity conditions, than the weak convergence approach applied in our work.

2. Statement of the problem and auxiliary results

Suppose that we observe n independents inhomogeneous Poisson processes $X^n = \left(X_1, \ldots, X_n\right)$, where $X_j = \Big(X_j\left(t\right), t \in \mathcal{R}\Big)$ are trajectories of the Poisson processes with the mean function $\Lambda\left(t\right) = \mathbb{E}X_j\left(t\right) = \int_{-\infty}^{t} \lambda\left(s\right) \mathrm{d}s$. Here $\lambda\left(\cdot\right) \geq 0$ is the corresponding intensity function.

Let us remind the construction of GoF test of Cramér–von Mises type in the case of simple null hypothesis. The class of tests $\left(\bar{\Psi}_n\right)_{n\geq 1}$ of asymptotic size $\varepsilon \in (0, 1)$ is

$$\mathscr{K}_\varepsilon = \left\{\bar{\Psi}_n \ : \qquad \lim_{n\to\infty} \mathbb{E}_0 \bar{\Psi}_n = \varepsilon\right\}.$$

Suppose that the basic hypothesis is simple, say, $\mathscr{H}_0 \ : \ \Lambda\left(\cdot\right) = \Lambda_0\left(\cdot\right)$, where $\Lambda_0\left(\cdot\right)$ is a known continuous function satisfying $\Lambda_0\left(\infty\right) < \infty$. The alternative is composite (non parametric) $\mathscr{H}_1 \ : \ \Lambda\left(\cdot\right) \neq \Lambda_0\left(\cdot\right)$. Then we can introduce the Cramér–von Mises (C–vM) type statistic

$$\tilde{\Delta}_n = \frac{n}{\Lambda_0(\infty)^2} \int_{\mathcal{R}} \left[\widehat{\Lambda}_n(t) - \Lambda_0(t)\right]^2 \mathrm{d}\Lambda_0(t),$$

where $\widehat{\Lambda}_n(t) = \frac{1}{n}\sum_{j=1}^{n} X_j\left(t\right)$ is the empirical mean of the Poisson process. It can be verified that under $\mathscr{H}_0$ this statistic converges to the following limit:

$$\tilde{\Delta}_n \Longrightarrow \Delta \equiv \int_0^1 W(s)^2 \mathrm{d}s,$$

where $W\left(s\right),\ 0 \leq s \leq 1$ is a standard Wiener process. Therefore the C–vM type test $\tilde{\psi}_n\left(X^n\right) = 1\!\!1_{\{\tilde{\Delta}_n > c_\varepsilon\}}$ with the threshold c_ε defined by the equation $\mathbb{P}\left\{\Delta > c_\varepsilon\right\} = \varepsilon$ belongs to $\mathscr{K}_\varepsilon$. This test is *asymptotically distribution free* (ADF) (see, e.g., Dachian and Kutoyants, 2007). Remind that the test is called ADF if the limit distribution of the test statistic under hypothesis does not depend on the mean function $\Lambda_0\left(\cdot\right)$.

Let us consider the case of the parametric null hypothesis. It can be formulated as follows. We have to test the null hypothesis

$$\mathscr{H}_0 \ : \ \Lambda\left(\cdot\right) \in \mathcal{L}\left(\Theta\right) = \Big\{\Lambda_0\left(\vartheta, t\right),\ \vartheta \in \Theta,\ t \in \mathcal{R}\Big\},$$

against the alternative $\mathscr{H}_1 \ : \Lambda\left(\cdot\right) \notin \mathcal{L}\left(\Theta\right)$. Here $\Lambda_0(\vartheta, \cdot)$ is a known mean function of the Poisson process depending on some finite-dimensional unknown parameter $\vartheta \in \Theta \subset \mathcal{R}^d$. Note that under $\mathscr{H}_0$ there exists the *true value* $\vartheta_0 \in \Theta$ such that the mean of the observed Poisson process $\Lambda\left(t\right) = \Lambda\left(\vartheta_0, t\right), t \in \mathcal{R}$.

The C–vM type GoF test can be constructed by a similar way. Introduce the normalized process $\bar{u}_n(t) \equiv u_n\left(t, \bar{\vartheta}_n\right) = \sqrt{n}\left(\widehat{\Lambda}_n(t) - \Lambda_0(\bar{\vartheta}_n, t)\right),\ t \in \mathcal{R}$. Here $\bar{\vartheta}_n$ is some estimator of the parameter ϑ, which is (under hypothesis $\mathscr{H}_0$) consistent and asymptotically normal $\sqrt{n}\left(\bar{\vartheta}_n - \vartheta_0\right) \Longrightarrow \xi$.

The corresponding C–vM type statistic can be

$$\bar{\Delta}_n = \frac{n}{\Lambda_0(\bar{\vartheta}, \infty)^2} \int_{\mathcal{R}} \left(\widehat{\Lambda}_n(t) - \Lambda_0(\bar{\vartheta}_n, t)\right)^2 \mathrm{d}\Lambda_0(\bar{\vartheta}_n, t)$$

Then, under null hypothesis $\mathscr{H}_0$, we can verify the convergence

$$\begin{aligned}
\bar{u}_n(t) &= \sqrt{n}\left(\widehat{\Lambda}_n(t) - \Lambda_0(\vartheta_0, t)\right) + \sqrt{n}\left(\Lambda_0(\vartheta_0, t) - \Lambda_0(\bar{\vartheta}_n, t)\right)\\
&= W_n\left(t\right) - \langle\sqrt{n}\left(\bar{\vartheta}_n - \vartheta_0\right), \frac{\partial \Lambda_0(\vartheta_0, t)}{\partial \vartheta}\rangle + o\left(1\right)\\
&\Longrightarrow W\left(\Lambda_0(\vartheta_0, t)\right) - \langle\xi\left(\vartheta_0\right), \dot{\Lambda}_0\left(\vartheta_0, t\right)\rangle.
\end{aligned}$$