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Role of memory effect in the evolution of cooperation based on spatial prisoner's dilemma game

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A B S T R A C T

We investigate the impact of memory effect on the evolution of cooperation in the spatial prisoner's dilemma game, in which each player will record his own strategies during the previous M game rounds (here, M is also named as the memory length). At each time step, each individual will update his current strategy according to the Fermi-like probability which will be multiplied by a pre-factor, and this factor will be correlated with the fraction of previous strategy states identical with the individual strategy to be updated. The numerical simulation results demonstrate that the memory length will largely influence the cooperation level at the stationary state, and it is clearly shown that the intermediate value of M will optimally favor the emergence of cooperation and the dynamical evolution, and characteristic patterns also support these conclusions. In addition, we depict the full cooperation phase diagrams and find that the cooperation region will be broadened under the case of moderate M values. The current results also indicate that the limited memory may be enough for us to design the effective promotion mechanism and further understand the emergency of cooperation taking place upon many networked populations.

1. Introduction

The cooperative strategy is often not a dominant strategy in the biological evolutionary game since it enhances the adaptability of the opponent and reduces its own evolutionary advantage [1,2], whereas the collective cooperation is still widespread inside animal and human societies. Thus, how to understand the evolution of cooperation within the selfish population has become an active topic in the field of social science, behavioral science, biology and physics, and even becomes one of 125 puzzling scientific problems listed by *Science* in 2009 and attracts a great deal of concern within the academic communities [3].

At present, the evolutionary game theory provides a powerful framework to explain the emergence of cooperation within the real-world population [4–6]. Among them, several mechanisms including kin selection, direct and indirect reciprocity, group selection, spatial or network reciprocity, have been identified as the effective schemes to promote the level of cooperation [9]. In particular, Nowak and May [10] seminally utilized the spatial lattice to explore the evolution of cooperation, and found that the lattice is beneficial for cooperators to form the cooperative clusters to defend the invasion of defectors. After that, various mechanisms, such as reward [11,12], punishment [13,14], utility coupling or weighting [15,16], individual mobility [17,18], reputation mechanism [19,20] and so on, have been added into the spatial reciprocity to discuss whether the cooperation can be further promoted (For the sake of knowing the state of art on the evolutionary game theory, we recommend the readers to refer to several comprehensive reviews [21–23].)

Recently, the progresses of network science have greatly enriched the recognition of complex systems [24,25], in which the small-world effect and scale-free properties have been found persuasively; that is, the network topology exhibits the strong inhomogeneity, unlike the traditional cases including the regular, totally random or well-mixing topologies. Henceforth, it is unarguably demonstrated that the complex topology will further enhance the fraction of cooperators within the networked population [26,27] and extensive works [28–42] are devoted to exploring the cooperation dynamics on complex networks. In particular, Refs. [43,44] reviewed the latest development in the field of evolutionary cooperation dynamics on complex networks.

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As a Chinese saying goes that the past experience, if not forgotten, is a guide for the future. Thus, an agent may refer to his previous strategies during the future decision making so that the best strategy will be chosen. However, like most previous works, the memory effect has not been paid much attention during the evolution of cooperation. Wang et al. [45] proposed a memory-based snowdrift game taking place on networks, which include lattices and scale-free networks, and they discovered the transitions of spatial patterns and step-like frequency of cooperation on lattices as the payoff parameters increase. While for scale-free networks, it is found that high degree nodes taken over by cooperators lead to a high cooperation level in the memory-based snowdrift game, and the non-monotonous cooperation behavior has been observed. In addition, we had ever presented an improved fitness evaluation mechanism with memory in spatial prisoner's dilemma game on regular lattices [46], which just integrates the payoff in the previous game round (i.e., one-step memory effect) into the current fitness calculation and does not consider the evolution of strategy states in the past multiple game rounds, and extensive numerical simulations indicate that the memory effect can substantially promote the evolution of cooperation for the spatial PDG as the proportion of payoff addition into the fitness of the current step increases.

In this letter, we further consider the role of memory effect in the evolution of cooperation based on the spatial prisoner's dilemma game (PDG), where we investigate the impact of memory effects involving multiple game rounds on the evolution of cooperation and the fraction of adopted strategies same as that of the current round in the past multiple game rounds is used to modify the Fermi–Dirac strategy update probability function, and the modification of Fermi-like probability is greatly different from previous work [46]. In addition, the current work is partially inspired by Challet and Zhang [47,48], who presented the so-called the minority game in which agents may make decisions exclusively according to the common information stored in their memories. It is clearly demonstrated through the extensive numerical simulations that the intermediate memory length is the most beneficial to promote the level of cooperation.

The rest of this letter is structured as follows. We firstly illustrate the game model with limited memory length in detail in Section 2, and then the extensive numerical simulation results are provided in Section 3. At last, in Section 4, we end this letter with some concluding remarks and point out the potential problems in the future.

2. The spatial game model with limited memory length

The system evolves on a spatial $L \times L$ regular lattice with a periodic condition, which accommodates $N = L^2$ players and each player is located at the intersection. Initially, each player will be designated as a cooperator (C) or defector (D) with the equal probability (i.e., one half of players are cooperators and the second half of ones are defectors within the population). After that, each player will accumulate the payoff through the interactions with all nearest neighbors according to the prisoner's dilemma game model; In this game model, the cooperator will obtain the reward (R) or sucker's payoff (S) if he plays with another cooperator or defector, while the defector will get the temptation (T) to defect or the punishment (P) provided that his opponent is one cooperator or defector, respectively. The payoff ranking condition $T > R > P > S$ must be satisfied for the prisoner's dilemma game. In general, we use the weak prisoner' dilemma model used by Nowak and May [10], where the parameter setup is set to be $R = 1$, $1 < T \le 2$ and $P = S = 0$ and the strict prisoner's dilemma game behavior can be totally reproduced under this scenario, but the only game model parameter T is considered here.

Then, one randomly chosen player (say, player x) will make a decision about the strategy (i.e., s_x) in the next game round, and he will randomly pick up a nearest neighbor (say, player y) and try to imitate this neighbor's strategy (i.e., s_y) with the following Fermi-like probability

$$Prob(s_x \leftarrow s_y) = w_x \frac{1}{1 + \exp[(P_x - P_y)/K]}, \quad (1)$$

where P_x and P_y denote the total payoff of player x and y, K implies the uncertainty of strategy adoption during the game playing [49], and the pre-factor w_x of player x is correlated with the fraction of strategies identical with the current strategy within the previous M game rounds. On the one hand, the Fermi function means that if payoff of player y is higher (lower) than that of player x, then player x will change his strategy in the next game step or round with probability approaching one (zero). On the other hand, however, player x may change his strategy with probability $\frac{1}{2}$ if payoffs of both players are equal (which happens in both the Nash and the optimal equilibria), that is, player x makes a decision like tossing a coin under this scenario. Thus, the novelty of the current work lies in the modification of Fermi-like probability function by comparing the current strategy state with the previous strategies during the past M game rounds, and pre-factor $w_x \le 1$ is introduced to modify this probability, which is the key for the Fermi-like procedure to obtain the expected behavior underlying the prisoner's dilemma.

Furthermore, the nearest neighbor y obtains the total payoff P_y with the same procedure as player x. For the simplicity, w_x is calculated as follows,

$$w_x = w_{max} - (w_{max} - w_{min}) \frac{n_x}{M}, \quad (2)$$

where w_{max} and w_{min} denote the maximum and minimum possible impact of memory behavior on the evolution of cooperation, M means the memory length for each player and its impact on the behavior of cooperation will be checked here; that is, each player will only record the strategy states of the previous M game rounds; meanwhile, n_x records the number of rounds that player x has adopted the current strategy in the past M game rounds, and thus w_x will always lie between w_{min} and w_{max}. If a player adopts the identical strategies during the previous M steps (i.e., $n_x = M$), w_x will be set as the minimum value w_{min}, that is, player x is willing to hold the current strategy with a higher probability and but not to change it; otherwise w_x will take the maximal value w_{max} if he always adopts the opposite strategy (i.e., $n_x = 0$). In order to avoid the frozen state, w_{min} will be set to 0.1 as the minimal scaling factor while w_{max} is assumed to be 1 without loss of generality. Therefore, the change in the variable n_x may become the driving force of the evolution of cooperation. In equilibrium, players should not deviate and then the probability of strategy change plunges as n_x tends to M. However, provided that a player has changed his/her strategy many times during the previous rounds, the cooperation would arise faster by 'resetting the memory' and then M might dominate the evolution of cooperation in the current model.

Finally, each player will have one chance on average to update his current strategy within one time step inside the whole population so as to proceed the evolution of cooperation, and a full Monte-Carlo simulation (MCS) step (which also means one game round, that is, one time step denotes one game round and these two terms can be interchangeably used in this letter) will be completed if the above-mentioned sub-steps have been completed. The system will arrive at the stationary state after some temporary steps are discarded. In our current simulation setup, the total MCS steps are up to 50000 (if unstated clearly) and the stationary state

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