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Peregrine solitons and gradient catastrophes in discrete nonlinear Schrödinger systems

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ABSTRACT

In the present work, we examine the potential robustness of extreme wave events associated with large amplitude fluctuations of the Peregrine soliton type, upon departure from the integrable analogue of the discrete nonlinear Schrödinger (DNLS) equation, namely the Ablowitz–Ladik (AL) model. Our model of choice will be the so-called Salerno model, which interpolates between the AL and the DNLS models. We find that rogue wave events essentially are drastically distorted even for very slight perturbations of the homotopic parameter connecting the two models off of the integrable limit. Our results suggest that the Peregrine soliton structure is a rather sensitive feature of the integrable limit, which may not persist under "generic" perturbations of the limiting integrable case.

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1. Introduction

The study of phenomena associated with extreme events and rogue (or freak) waves has gained substantial traction over the last few years [1–4]. This can largely be attributed to the development of experimental settings in a variety of fields where the relevant coherent structures can be systematically created and observed. These fields range from superfluid helium [5] to hydrodynamics [6–8], and from nonlinear optics [9–14] and plasmas [15] to Faraday surface ripples [16] and parametrically driven capillary waves [17]. Experimental efforts have, in part, been motivated by – and also inspired – numerous theoretical investigations, mainly concerning variants of the nonlinear Schrödinger (NLS) equation. The theoretical activity has now been summarized in many reviews [18–20] and books [1–4].

One of the significant aspects of the investigation of extreme wave events has to do with the structural form that these events assume, and perhaps especially with their robustness in NLS and related models. The seminal works of Peregrine [21], Kuznetsov [22], Ma [23], and Akhmediev [24], as well as of Dysthe and Trulsen [25], have provided a framework of study of relevant coherent structures, either periodic in space (such as the Akhmediev breather) or periodic in time (such as the Kuznetsov–Ma breather) or, most notably, localized in space-time, as the Peregrine soliton [21]. A question then emerges about whether these entities survive model perturbations and/or emerge under generic classes of initial conditions. Admittedly, the latter question has only been partially addressed. For instance, in a class of perturbations involving Hirota-model variants (such as third-order dispersion and self-steepening terms), a perturbed, still algebraically decaying variant of the Peregrine soliton was obtained (however its persistence was not ensured to all orders in the perturbation) [26]. Moreover, adiabatic approximations [27] and perturbed inverse scattering approaches [28] have considered the stability of Kuznetsov–Ma (KM) solitons indicating their potential robustness against dispersive but non-robustness against dissipative perturbations. A different perspective on the emergence of localized phenomena in space-time was given by the work of [29], where it was argued that the proximity of such solutions to chaotic states (in more elaborate, non-integrable models) appears to increase the occurrence of extreme events. Other works have focused on the stability properties of solutions [30–32]; however, there is an ambiguity associated with the time-dependent nature of the solutions. A natural setup for performing stability studies is, arguably, the Floquet analysis of the time-periodic KM breather solution [33].

In a recent work [34], a different type of "genericity" of these solutions was considered: the emergence of extreme events stemming from simple – yet typical – Gaussian initial data, under a phenomenon called gradient catastrophe that has been explained in the pioneering work of [35]. In particular, it was proposed that in the semiclassical limit of the NLS model, such initial data will lead to the formation of an array of essentially identical (up to

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small corrections) Peregrine soliton-like structures, which emerge at the poles of the so-called tritronquée solution of the Painlevé I equation. The importance of these findings is underscored by the fact that very recently the universality of this emergence of the Peregrine soliton in the semiclassical focusing dynamics of the NLS model has been manifested experimentally [36]. Furthermore, recently, such Peregrine waveforms were also found to spontaneously emerge as a result of the interaction of dispersive shock waves [37].

Our aim in the present work is to explore the relevance of Peregrine-soliton type solutions in spatially discrete systems, i.e., in nonlinear dynamical lattices. There has been an amount of work in this context as well. In particular, it has been established that a Peregrine-like solution, strongly reminiscent of its continuum sibling exists [38] in the context of the completely integrable discrete version of the NLS equation, the so-called Ablowitz–Ladik (AL) model [39,40]. In fact, subsequent work has established the systematic construction of higher-order such solutions [41]. However, it is also well-known that while the AL model is useful for the consideration of numerous perturbative calculations involving single discrete solitons [42], their stability [43] and their collisional dynamics [44], it is not of direct relevance to experimental settings. On the contrary, the quintessential discrete model of relevance, both to nonlinear optics (in the context of arrays of coupled waveguides) and to atomic Bose–Einstein condensates (BECs) confined in optical lattices, is the discrete nonlinear Schrödinger (DNLS) equation [45]. Hence, our considerations herein will involve departing in a systematic way from the AL model and approaching the DNLS one. This will be done through the Salerno model [46], interpolating between the two limits.

We consider a Gaussian initial profile (as a generic waveform) and examine a two-parametric variation. In particular, on the side of varying the initial condition parameters, we examine the effect of changing the variance of the initial condition (IC). Here, using a large variance places us within the so-called semi-classical regime [35], where we may expect analogously to the continuum case of [34] to observe Peregrine soliton like structures. On the other hand, at the level of varying the model, we consider changes of a homotopic parameter as extending from the AL limit all the way to the DNLS one, and examine a wide variety of cases in between. Our main observation is that at the AL limit, we identify Peregrine structures and even a space-time evolution featuring the emergence of a "Christmas-tree"-like pattern, analogous to the continuum case [34], as an apparent discrete emulation of the gradient catastrophe phenomenology of [35]. Nevertheless, this appears to be – in some sense – a singular case, in that as soon as we depart from this integrable limit, the prevalent dynamical structures appear to consist of persistent or breathing in time discrete solitonic entities (discussed at length in the context of DNLS models [45]), rather than of Peregrine-like patterns. It is intriguing to point out that a similar conclusion (a propensity towards freak waves near the integrable limit) had emerged through the important statistical analysis of Ref. [47]. In fact, our observations lead us to conjecture that no direct analogue of the Peregrine soliton exists in the DNLS model, although one exists in its continuum limit, as well as in its integrable discrete sibling. A dynamical systems analysis that would tackle this persistence problem would be of paramount importance for future work.

Our presentation is structured as follows. In section 2, we present the relevant mathematical model(s) and the corresponding prototypical solutions. In section 3, we establish the corresponding numerical results and comment on the relevant observations. Finally, in section 4, we present a summary of our findings and provide some suggestions for future work.

2. The model

The model of interest originates from the focusing NLS equation, written in dimensionless form as follows:

$$i\partial_t u = -\frac{1}{2}\partial_x^2 u - |u|^2 u, \tag{1}$$

where $u(x,t) \in \mathbb{C}$ is the wave function. Next, discrete realizations of Eq. (1) can be obtained, e.g., by replacing the (continuous) dependent variable $u(x,t)$ with $u_n(t) \doteq u(x_n,t)$ ($x_n = -L + nh$ with L the grid's half-width) and the second partial derivative with its second-order accurate centered finite difference operator. This way, we obtain the discrete NLS (DNLS) equation:

$$i\dot{u}_n = -\frac{1}{2h^2}\left(u_{n+1} - 2u_n + u_{n-1}\right) - |u_n|^2 u_n, \quad n \in \mathbb{Z}, \tag{2}$$

where overdot stands for differentiation with respect to time and, hereafter, we will set the lattice spacing $h = 1$ [48]. Details on the derivation and physical origin of the DNLS, e.g., in coupled optical waveguides and in BECs confined in optical lattices, as well its discrete soliton solutions, can be found in the review [45].

A different discretization of Eq. (1) can be obtained by discretizing the field value multiplying the square modulus in Eq. (2) as $u_n \doteq (u_{n+1} + u_{n-1})/2$. The resulting discrete lattice model is the AL model [39,40], which is of the form:

$$i\dot{u}_n = -\frac{1}{2}\left(u_{n+1} - 2u_n + u_{n-1}\right) - \frac{1}{2}|u_n|^2\left(u_{n+1} + u_{n-1}\right). \tag{3}$$

To interpolate between the DNLS and the AL models, we introduce a real parameter $\mu \in [0,1]$. Then, we can write the following "tunable" discrete lattice system:

$$\begin{aligned} i\dot{u}_n = &-\frac{1}{2}\left(u_{n+1} - 2u_n + u_{n-1}\right) - \mu|u_n|^2 u_n \\ &-\frac{1}{2}\left(1-\mu\right)|u_n|^2\left(u_{n+1} + u_{n-1}\right), \end{aligned} \tag{4}$$

which corresponds to the DNLS and AL models for $\mu = 1$ and $\mu = 0$, respectively. This generalized Salerno model [46] of Eq. (3) will be the focal point of our subsequent numerical investigations.

Here, it should be noted that the AL model supports rational solutions of the rogue wave type: in particular, the first-order such rational solution of the AL system is of the form [38]:

$$u_n = U_n e^{i\phi}, \quad U_n(t) = \left(\frac{4q\left(1+q^2\right)\left(1+2iq^2 t\right)}{1 + 4q^2n^2 + 4q^4\left(1+q^2\right)t^2} - q\right)e^{iq^2 t}, \tag{5}$$

with q and ϕ being a real background amplitude and (arbitrary) phase, respectively. We will compare our findings to this solution, especially so in computations associated with the AL limit. In that light, the background amplitude q will be utilized as a fitting parameter to obtain the "best-fit" Peregrine soliton.

Our goal is to study the initial value problem (IVP) which consists of Eq. (4) (for various values of $\mu \in [0,1]$) and Gaussian initial data, of the form:

$$u_n(t=0) = e^{-n^2/2\sigma^2}, \tag{6}$$

where σ characterizes the Gaussian's width. In a vein reminiscent of the work of Ref. [34], we are interested in identifying parametric regimes of both σ and μ, such that extreme events (or fundamental solitons) can be obtained at the discrete level.

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