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Chaos and reverse transitions in stochastic resonance

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ABSTRACT

Stochastic resonance is a phenomenon that a weak signal can be amplified and optimized by the assistance of noise in bistable system. There is still not enough research on the mutual interplay among system, noise and signal. In this paper, we study the role of every parameter in nonlinear transfer and discover chaos phenomenon in stochastic resonance. To measure the influence of chaos, a trajectory decision function was proposed. Based on this function, we found two forms of stochastic resonance, clockwise resonance and counterclockwise resonance.

1. Introduction

The concept of stochastic resonance (SR) was originally put forward by Roberto Benzit et al. [1] for explaining the periodicity of ice ages in 1981. They used the mechanism of over-damped particles moving in double-well to illustrate the mechanism of stochastic resonance. The phenomena have been researched extensively in theory and experiment. McNamara et al. (1989) [2] proposed a two-state model which can be regarded as an adiabatic approximation to any continuous bistable system. Hanggi et al. (1991) [3] extends the linear response theory of statistical equilibrium systems to stochastic processes. Collins (1995) et al. [4,5] proposed the concept of aperiodic stochastic resonance (ASR), based on which various applications emerged [6–9]. Stocks (2000) et al. [10,11] studied the effect of noise on the suprathreshold signals based on a summing network of N threshold devices, which was termed as suprathreshold stochastic resonance (SSR). By an appropriate choice of threshold level, noise can also optimize the detection of suprathreshold signals [12–15]. Steven Kay [16] explained the reason of detector performance improved by adding noise.

Classical stochastic resonance mainly focuses on periodic signals and Gaussian white noise. The effect of different types of noise, such as levy noise, Poisson white noise, fractional Gaussian noise and colored noise, is further studied [17–21].

The resonance ability of bistable system can be enhanced by tuning system parameters [21–24]. In addition to the traditional bistable stochastic resonance system, piecewise bistable SR and periodic potential SR are proposed [25,26].

Numerous contributions to stochastic resonance have appeared and the phenomena are well explained under the respective hypothetical conditions. However, these theories do not adequately explain the nonlinear dynamic behavior of stochastic resonance systems. Therefore, this paper intends to find a straightforward approach to explain the mutual interplay between the transfer of power among the system $x(t)$, the sources of noise $\Gamma(t)$, and the signal $s(t)$. We examined how each parameter acts on resonance output and found two forms of stochastic resonance based on the function we put forward, i.e., clockwise resonance and counterclockwise resonance. Normally the particles in the plane move in a clockwise direction, and the system output signal has the same phase as the original signal. But under certain conditions, we found the particles can also move in a counterclockwise direction, which is reverse of the former. And the system output signal is exactly opposite in phase to the original signal. Furthermore, we found a tipping point that particles enter chaos status and relationship between the point and stochastic resonance system parameters.

2. System model

It is known that the SR phenomenon can be expressed by the Langevin equation, which is governed by Eq. (1).

$$\begin{aligned} \frac{dx}{dt} &= -\frac{\partial V x)}{\partial x} + u(t) \\ u(t) &= s(t) + \Gamma(t) \end{aligned} \tag{1}$$

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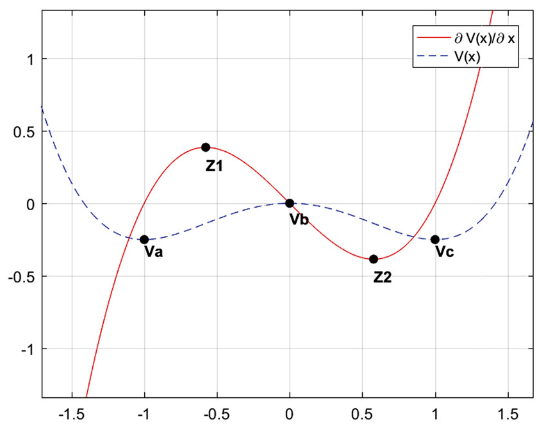

Fig. 1. The sketch of V(x) and $\frac{\partial V x)}{\partial x}$ when *a*=*1*, *b*=*1*.

where *V(x)* is the nonlinear potential function and *u(t)* is the input signal *s(t)* mixed with noise $\Gamma(t)$, which is the Gaussian white noise with intensity D, Eq. (2).

$$\begin{aligned} &< \Gamma(t) >= 0 \\ &< \Gamma(t), \Gamma(t') >= 2D\delta(t-t') \end{aligned} \tag{2}$$

For the bistable potential, *V(x)* can be described as Eq. (3).

$$V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4, a > 0, b > 0 \tag{3}$$

By deriving the partial derivative of the function *V(x)*, we can get Eq. (4), which represents the change rate of potential function.

$$\frac{\partial V(x)}{\partial x} = -ax + bx^3, a > 0, b > 0 \tag{4}$$

Fig. 1 is sketch of potential function *V(x)* and partial derivative $\frac{\partial V x)}{\partial x}$. The inflection points of potential function *V(x)* located at $V_a(-\sqrt{\frac{a}{b}}, -\frac{a^2}{4b})$, $V_b(0,0)$, $V_c(+\sqrt{\frac{a}{b}}, -\frac{a^2}{4b})$. The inflection points of $\frac{\partial V x)}{\partial x}$ located at $Z_1(-\sqrt{\frac{a}{3b}}, +\sqrt{\frac{4a^3}{27b}})$, $Z_2(+\sqrt{\frac{a}{3b}}, -\sqrt{\frac{4a^3}{27b}})$.

If $u(t) = \frac{\partial V(x)}{\partial x} = -ax + bx^3$ is satisfied, then $\frac{dx}{dt} \equiv 0$, that is to say, *x* is stable and does not change over time under this assumption. Substituting Eq. (3) into Eq. (1), the governing Eq. (5) can be obtained.

$$\frac{dx}{dt} = ax - bx^3 + s(t) + \Gamma(t) \tag{5}$$

If we consider the overdamped motion of a Brownian particle in a symmetric double well in the presence of noise and periodic forcing, Eq. (5) can be written as a generic model Eq. (6).

$$\frac{dx}{dt} = ax - bx^3 + A\cos(2\pi ft) + \Gamma(t) \tag{6}$$

Due to the non-linear and non-autonomous features of Eq. (6), it is difficult to obtain its exact solution expression. To obtain numerical solution with high-precision [26–28], we consider fourth-order Runge–Kutta algorithm, Eq. (7).

$$\begin{aligned} &x_{n+1} = x_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ &k_1 = h(ax_n - b{x_n}^3 + u_n) \\ &k_2 = h[a(x_n + \tfrac{k_1}{2}) - b(x_n + \tfrac{k_1}{2})^3 + u_n] \\ &k_3 = h[a(x_n + \tfrac{k_2}{2}) - b(x_n + \tfrac{k_2}{2})^3 + u_{n+1}] \\ &k_4 = h[a(x_n + k_3) - b(x_n + k_3)^3 + u_{n+1}] \end{aligned} \tag{7}$$

Where u_n represent the *n*th sample of mixed input signal, x_n is the *n*th sample of output *x*, *h* is the interval size. The following dynamic behavior analysis of bistable potential is based on the Runge–Kutta algorithm.

3. Numerical simulation

It can be seen from the Eq. (6) that the system output *x* is related to several parameters. The Eq. (6) can be abbreviated as Eq. (8) in the method of numerical calculation.

$$x = F(a, b, h, fs, A, f, \Gamma) \tag{8}$$

Where *a* and *b* are the potential parameters, *h* is an interval size which usually equals to *1/fs*, *fs* is the sample frequency, *A* and *f* are the key parameters of the input signal, Γ is the mixed noise. In order to study the characteristics of stochastic resonance in depth, we individually analyze each parameter by simulation to get the results of regularity.

3.1. Potential parameter a, b

The *a* and *b* are the barrier parameters of the bistable potential, which determine the shape of potential well and projection distribution of particles. Fig. 2 shows the projection of the identical cosine signal on different structures. The parameters of the cosine signal are $A = 1$, $f = 0.01$, and sample frequency *fs*=2. In Fig. 2, A.1, B.1, C.1 are the input–output diagram, the x-axis represents the output, the y-axis is the input signal, and the red curve is partial derivative of potential *V(x)*. A.2, B.2, C.2 are the corresponding output waveform. Panel A shows the particles oscillate inside the well along the red curve. Panel B shows the case where particles move across the obstacle to achieve inter-well oscillation. Panel C shows the particles achieve inter-well oscillation and the oscillation is period-3, which means that chaos will occur in the system [29,30]. Because the bistable stochastic resonance system is nonlinear and non-autonomous, it is found in subsequent experiments that the system does produce chaos under certain conditions.

3.2. Interval size h and sampling frequency fs

The *h* is the interval size, which essentially characterizes the calculated error. In the Runge–Kutta algorithm, the interval size *h* affects *a*, *b*, and input signal *u* at the same time. Some scholars try to modify the interval size h to improve the output SNR [31]. For the sampling frequency *fs*, according to Shannon sampling theorem, in order to restore the analog signal without distortion, *fs* should not be less than twice the highest frequency in the spectrum of the analog signal. Both of these two parameters affect accuracy. In this paper, the sampling frequency *fs* is set as more than 100 times of the signal frequency. Fig. 3 shows the output of the identical cosine signal sampled with different sampling frequency. In the figure, A.1, B.1 are the input–output diagram, the x-axis represents the output, the y-axis is the input signal. A.2, B.2 are the corresponding output waveform. Obviously, A.1 is in a bifurcated state and A.2 is not.

3.3. Pure periodic signal

To further observe the movement of particles in one potential well or between the wells, we put the signals which have the same period but with different amplitude into the identical bistable system, and observe the output track. Fig. 3 illustrates what happens if amplitude *A* of the signal increases linearly, where *a*=1, *b*=1, *f*=0.01, *fs*=1, *h*=1.

It is clear that when input signal $A < \sqrt{\frac{4a^3}{27b}}$ the particles oscillate at one side, just as Fig. 4A shows. When $A \geq \sqrt{\frac{4a^3}{27b}}$ the particles can oscillate at both sides in a clockwise direction, just as Fig. 4B shows. We found that there exists a special threshold