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Orbital angular momentum of channeling radiation from relativistic electrons in thin Si crystal

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ABSTRACT

We propose to use channeling radiation (CR) from relativistic electrons as a source of high energy twisted photons in the MeV range. We calculate numerically the orbital angular momentum (OAM) of radiation produced by electrons with the energies $155 \div 2500$ MeV for the axial and planar channeling in the thin Si crystal. We obtain that the average OAM of CR in this case is approximately $1 \div 6\hbar$ per photon with the photon energies about $1 \div 2$ MeV.

1. Introduction

Current research on photons involves the orbital angular momentum (OAM) as a new degree of freedom. The studies on the OAM of light that were started more than a century ago [1–4] have been revived rather recently by the seminal paper [5]. At present, the key technologies to generate, manipulate, and detect the photons carrying OAM (i.e., twisted photons) are well known both in the optical [6–8] and the radio frequency ranges [9]. The optical twisted photons are used for encoding of optical signals to increase the data transfer rate [6] and for the quantum entanglement of photons in their OAM [7]. Manipulation with the radio beams carrying OAM reveals a new field of research in radio astronomy [10]. The optical vortices in the EUV regime are used in atomic spectroscopy and nanostructure manipulations [11]. The possibility to generate photons with the energy of the order of several MeV that carry OAM paves the way for new research in photonuclear reactions and provides the new tools in nuclear physics.

Generation of high energy (above MeV) twisted photons is still a problem. According to [12], the Compton backscattering of 1.2 eV twisted photons from a laser beam by 5 GeV electrons theoretically results in twisted photons with energies up to 420 MeV. This process, however, is hard to use as a brilliant source of twisted photons. It is of second order, i.e., its cross-section is proportional to the fine-structure constant squared. The first order processes to produce the twisted photons are also known. In the theoretical paper [13], the idea to employ the helical undulator as a source of hard X-ray twisted photons was suggested. In the experimental work [14], the dual (helical-linear) undulator scheme was used to generate the twisted photons with energy 99 eV and OAM $\pm 1\hbar$. However, generation of MeV photons by undulators requires a multi-GeV initial electron beam [15]. In contrast to undulators, the high strength crystalline fields allow one to generate the above-MeV photons by channeling of sub-GeV charged particles [16,17]. Channeling radiation (CR) of electrons in crystals possesses the angular divergence of the order γ^{-1} [18], where γ is the Lorentz factor, as compared with γ^{-2} (hereinafter all the angles are given in radians) for the Compton backscattering.

The radiation from relativistic electrons moving along a helical trajectory is known to possess OAM [19–21]. So, in the present paper, we calculate the average OAM per photon of CR produced by relativistic electrons in thin crystals and estimate the prospects for this radiation to be a source of several MeV twisted photons. Due to a complex pattern of channeled electron trajectories (Fig. 1), realistic CR is not the eigenfunction of the angular momentum projection operator but is a superposition of twisted photons (see for a mathematical definition, e.g., [22,23]). Thus, CR should be considered as an impure source of twisted photons.

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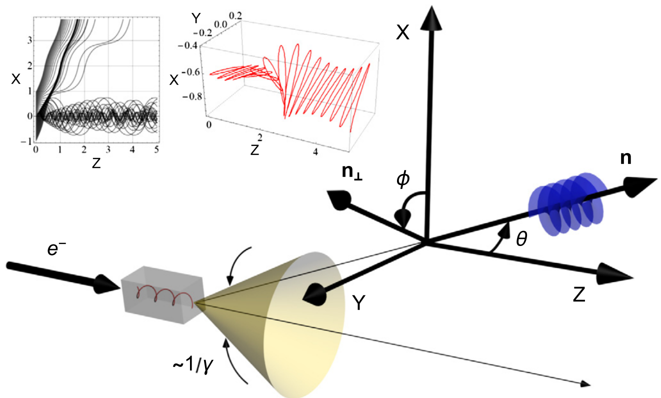

Fig. 1. Schematic view of the twisted photon production by channeling electrons. The channeling planes are parallel to the YZ plane, and the channeling axes are parallel to the Z axis. Here, $\boldsymbol{n}\,(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ is the unit vector pointing to the detector. The origin is chosen at the center of the electron beam. The typical trajectories for the planar (XZ) and axial (XYZ) channeling are depicted on the top.

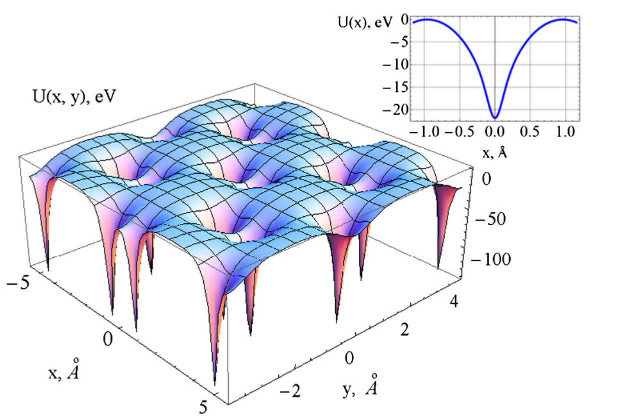

Fig. 2. The potential energy of an electron in the electric field of the system of ⟨110⟩ crystallographic axes (left) and (220) planes (right) [28]. The axial potential possesses the reflection symmetry with respect to the planes $x = 0$ and $y = 0$. The planar potential has the symmetry plane $x = 0$.

2. Methods

We consider the radiation of 155–2500 MeV electrons for (220) planar and ⟨110⟩ axial channeling in a thin Si crystal. The initial electron beam energies are taken as to comply with the accelerators parameters at INFN-LNF [24], SAGA-LS [25], and MAMI [26]. The direction of the initial velocity of electrons with respect to the atomic planes and axes is described in Fig. 1. The thickness of the crystal is set to be less than 10 μm to avoid the dechanneling effects [18,27]. The potential energy of channeled electrons in Si is calculated using the modified Doyle-Turner approximation [28] and presented in Fig. 2. In order to find the OAM of CR, we employ the semi-classical method of Ivanenko–Sokolov [29]. In fact, we use the equivalent expression for the OAM written in terms of the trajectories of radiating particles [21,30]. Since CR is concentrated in the cone with the opening of the order γ^{-1} [18], this method allows one to estimate the average OAM of CR per photon detected in this cone. The trajectories of channeled electrons are the solutions of the classical relativistic equations of motion [18]. We find the trajectories using Wolfram Mathematica code BCM-2 [31]. The process of multiple scattering can be neglected due to the small thickness of the Si crystal. The distribution of electrons is supposed to be uniform in the initial positions and the delta-function with respect to the initial velocities. The combination of the methods used is new and, to our knowledge, has never been applied before.

3. Trajectories of electrons at channeling and properties of CR

The trajectories of relativistic (with the energy 0.1–10 GeV) electrons channeling in the crystal are the solutions of the modified classical relativistic equation for the transverse motion [18]:

$$\frac{m}{\sqrt{1-(v_z^2/c^2)}}\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F} = -\nabla U, \tag{1}$$

where m is the electron mass, $\boldsymbol{F}$ is the force applied to the electron, and $U(x, y, z)$ is the potential energy of the electron moving along the crystal axis or near the crystal planes. The initial conditions for Eq. (1) are the initial electron coordinates and the projections of the initial electron velocity to the OX and OY axes: $x(0) = x_0$, $y(0) = y_0$ $v_x(0) = c\sqrt{1-\gamma^{-2}}\sin\theta_0\sin\phi_0$, $v_y(0) = c\sqrt{1-\gamma^{-2}}\sin\theta_0\cos\phi_0$.

The average energy of the photons [18] for the axial channeling is given by

$$\omega_{Ch} = 2\gamma^2\theta_c/a_s, \tag{2}$$

where $a_s = 0.299$ Å is the screening radius [18] for the Si crystal. Therefore, the average number of photons of CR becomes

$$N_{Ch} = Y_{Ch}/\omega_{Ch}, \tag{3}$$

where Y_{Ch} is the total yield calculated in Ref. [32]. We define Y_{Ch} as the total energy radiated by an electron during channeling in a crystal of a certain thickness.

It is well known [18] that $Y_{Ch} \sim \gamma^2$, $\omega_{Ch} \sim \gamma^{3/2}$, and $N_{Ch} \sim \sqrt{\gamma}$. In the case of ⟨110⟩ channeling of 255 MeV electrons in 5 μm Si, the average energy of the photons is approximately 2.6 MeV, Y_{Ch} is about 3.1 keV, and the average number of photons is 1.2×10^{-3} per incident electron. In the case of (220) channeling of 255 MeV electrons in 5 μm Si, it follows from the expression similar to (2) and given in [18] that the average energy of the photons is approximately 1.05 MeV, Y_{Ch} is about 0.4 keV, and the average number of photons is 3.8×10^{-4} per incident electron. The critical Lindhard angle $\theta_c \sim \gamma^{-1/2}$. For the 255 MeV electrons channeled in Si, $\theta_c \approx 10^{-3}$ rad for ⟨110⟩ channeling, and $\theta_c \approx 4 \times 10^{-4}$ rad for (220) channeling.

4. Orbital angular momentum of radiation from channeled electrons

The OAM of the electromagnetic field emitted by a relativistic particle in the wave zone reads [29,30]:

$$\tilde{\Lambda}^{\mu\nu} = \frac{2}{3}\frac{e^2}{c^5}\int \omega_\rho\omega^\rho\left(r^\mu v^\nu - r^\nu v^\mu\right)d\tau, \tag{4}$$

where r^μ, v^ν and ω^ρ are the four-dimensional coordinates, the velocity, and the acceleration of the particle, respectively. The particle charge is denoted as e, τ is the proper-time, and c is the speed of light.

In the case of planar channeling (Fig. 1), the expression for OAM (4) can be considerably simplified. The only nonzero component of OAM is the OY one:

$$\tilde{\Lambda}_y = \tilde{\Lambda}^{31} = \frac{2}{3}\frac{e^2}{c^5}\gamma^4\int {a_x}^2\left(xv_z - zv_x\right)dt, \tag{5}$$

where $\tilde{\Lambda}^{31}$ is the OY component of the OAM of CR, t is the laboratory time, $\boldsymbol{r}(x, y, z)$, $\boldsymbol{v}\left(v_x, v_y, v_z\right)$, and $\boldsymbol{a}\left(a_x, a_y, 0\right)$ are the three-dimensional radius-vector, the velocity, and the acceleration of the electron, respectively.