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Superfluidity of indirect excitons vs quantum Hall correlation in double Hall systems: Different types of physical mechanisms of correlation organization in Hall bilayers

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A B S T R A C T

Bilayer Hall systems can be divided into two groups—with and without tunneling of carriers across the barrier between layers. We demonstrate that these both classes differ in topology sense which leads to the distinct quantum Hall hierarchy. In the case of forbidden interlayer carrier tunneling we developed the Metropolis Monte Carlo simulation for an energy competition of the reentrant integer quantum Hall state against the superfluid Bose Einstein condensate of indirect excitons in double-layer 2D Hall systems, GaAs/GaAlAs/GaAs and b-graphene/hBN/b-graphene, with complementary layer filling, $\nu_{bot} + \nu_{top} = 1$. The resulted phase diagrams for both systems have been determined in consistence with the experimental data.

1. Introduction

Advances in Hall experiment and new techniques of ultrathin multilayer heterostructure manufacturing accelerated studies of interaction-induced quantum correlations in multiple planar Hall systems [43,50,28,33,32,48,47,9,8,27]. Double-layer Hall systems exhibit a different FQHE hierarchy in comparison to monolayer ones. To describe a variety of related experimental observations two different physical mechanisms in bilayer systems should be outlined. The first one, typical in bilayer graphene, corresponds to the interlayer tunneling of electrons which makes this bilayer system topologically different than the pair of monolayers [18,36]. Because of electron delocalization among layers, the bilayer graphene behaves as the unified material rather than a layered multicomponent system. However, a bilayer is still not perfectly 2D and the braid group defining correlations in the system is modified in comparison to a Hall monolayer [18]. The second mechanism manifests itself when the tunneling of electrons is precluded but two Hall systems are still coupled by the Coulomb interaction across a sufficiently thin insulating barrier. This interaction may also modify Hall correlations which distinguishes such coupled system from two independent monolayers. There are various practical realizations of both situations in multilayer graphene heterostructures [33,32,29,30,38] and in conventional GaAs bilayers [43,50,28,48,47,9,8,27]. With regard to differences between the indicated two situations it is especially instructive to revisit some pioneering bilayer experimental studies [10,46]. In these two papers it has been reported unexpected observation of FQHE at total filling $\nu_T = \frac{1}{2}$ of bilayer GaAs Hall systems and a series of other Hall features being not a sum of the Hall response of two independent layers. In Ref. [10] two closely located GaAs wells of width 18 nm were separated by thin GaAlAs barrier (of thickness 3.1 nm and 9.9 nm, for comparison), whereas in Ref. [46] the single wide (68 nm) GaAs well has been utilized to create two Hall system near well borders due to the repulsion of electrons with a level separation of symmetric electron-layers controlled by the carrier density. Both these papers supply rich information about interplay between various regimes of interlayer tunneling and interaction.

It should be noted that bilayer systems consisting on two monolayer graphene sheets separated by an insulating barrier have been studied formerly at various filling rates of both layers, with and without magnetic field, for variating insulator barrier thickness and out of the quantum Hall regime even up to room temperature [4,2,40]. In these papers it has been studied an extremely interesting problem of high-temperature counterflow superfluidity of indirect excitons [40] created by strong coupling of oppositely charged carriers by the Coulomb attraction across the barrier. In an

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effective model of the Peierls-type instability for nesting electron and hole Fermi surfaces in two parallel layers [40] the electron–hole strong pairing substitutes a weak Cooper phonon-mediated pairing of electrons in BCS superconductors [1], hence, the theoretically predicted critical temperature for exciton counterflow superfluidity may reach even room temperatures. It is, however, not demonstrated experimentally as of yet, because of disorder constraints limiting the predicted Costerlitz–Thouless transition [4,2]. Moreover, a wide study of drag effects in twin parallel closely located planar electron systems, with and without magnetic field presence, has been carried out also far from the quantum Hall regime [41,49], including two monolayer graphene sheets separated by very thin, even of few nm thickness of hBN barrier, in temperatures in the range 140–260 K with various configurations of layer charging [13]. In particular, it has been demonstrated that for perfectly opposite charging of both graphene layers the drag response is especially pronounced [13]. These drag and superfluid counterflow phenomena [13,41] are induced by strong interlayer electrical Coulomb coupling of carriers in both sheets in the regime when the tunneling of electrons (holes) across the barrier is forbidden [41,40], and the possible Hall correlations (if a perpendicular quantizing magnetic field is applied) are washed out by the temperature chaos. The role of the Coulomb interlayer interaction in the Hall regime can be characterized by the ratio $\frac{d}{l_B}$, because the Coulomb interlayer coupling $\sim \frac{e^2}{d}\left[\frac{1}{4\pi\varepsilon_2\varepsilon_0}\right]$ whereas the intra-layer electron interaction $\sim \frac{e^2}{l_B}\left[\frac{1}{4\pi\varepsilon_1\varepsilon_0}\right]$ (here, $\varepsilon_{1(2)}$ is the dielectric permittivity of the layer (barrier), $l_B = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length). For graphene sheets separated by hBN the electron tunneling is blocked even up to few nm for d [13] but for GaAs systems the GaAlAs barrier is less well insulating and to prevent tunneling must be thicker, over of dozen nm. Extremely interesting is, however, also the regime for bilayers with electron-tunneling admitted [46,10], which is the regime in the bilayer-graphene in fact [29,30,38,18,22,36]. The experiments with twin GaAs Hall systems in this regime [46,10] revealed a puzzling observation of incompressible IQHE/FQHE states at $\nu_T = 1, \frac{1}{2}$ and $\nu_{bot} = \nu_{top} = \frac{1}{2}, \frac{1}{4}$, respectively, being not a sum of Hall response of independent layers (the subscripts *bot* and *top* refer to bottom and top layer, respectively).

In the present letter we will describe two different model bilayer systems with different correlation organization induced by the change of the interlayer electron tunneling regime—with allowed or prohibited interlayer tunneling of carriers for still Coulomb strongly coupled layers.

In the case when electrons can hop between layers, as in the bilayer graphene or in two adjacent layers of 2DEG GaAs with small barrier allowing tunneling, the system behaves as a uniform Hall system (with total filling ν_T symmetrically divided for $\nu_{bot} = \nu_{top} = \nu_T/2$ counted per layer). The most important observation is that such a system has the different topology than a monolayer Hall plane because it is actually not an ideal 2D planar system as a monolayer is. This feature can be expressed in terms of a classical braid group defining the homotopy of the corresponding quantum system [3,20,21,51,17,24]. The braids can reside simultaneously in both layers here which causes a change in homotopy of braid trajectories in comparison to the monolayer case and results in different quantum Hall hierarchy. If, however, the barrier between layers blocks interlayer tunneling of electrons (for sufficiently thick insulating barrier or by creation of an additional barrier by an external field) then the homotopy of the monolayer type is restored and both layers behave as independent monolayer Hall planes though still coupled by Coulomb forces. The related homotopy phase transition induced by the change of the electron tunneling regime can be convincingly demonstrated experimentally by application of a vertical voltage perpendicular to the bilayer graphene sample. Such an electrical field can block hopping of electrons in one direction, which, however, completely blocks cyclotron trajectory hopping (trajectory is closed and must return), considerably changing the trajectory homotopy. Such an experiment [38] actually evidences FQHE hierarchy change from the bilayer to monolayer one [18].

When the interlayer hopping of carries is completely blocked a specific collective Hall response still manifests itself but at a specific complementary filling of layers only, $\nu_T = \nu_{bot} + \nu_{top} = 1$ (or other integer). In the following paragraph we will describe the first situation with interlayer tunneling admitted whereas the second situation with completely insulating barrier we will elaborate in the next one.

2. Bilayer Hall system with tunneling of electrons between layers

The bilayer graphene (or two closely adjacent GaAs layers with interlayer tunneling of electrons) creates a unified Hall system for which the filling rate can be counted as in 2D system, $\nu_T = \frac{N}{N_0}$, where N is the total number of electrons and N_0 is the degeneracy of 2D Landau level (LL) subbands, $N_0 = \frac{BSe}{h}$, with S—the surface of the sample (the surface of the single layer), B—the external magnetic field, $\frac{h}{e}$—the magnetic field flux quantum. Thus, at the total filling rate, ν_T, per each layer falls $\nu_{top} = \nu_{bot} = \nu_T/2$. It should be added that in the bilayer graphene one deals with additional spin-valley four-fold quasi-degeneracy (referred conventionally to as SU(4) degeneracy) of LL which splits it into four subbands due to Zeeman effect and valley inequivalence induced by magnetic field, imperfections or strain [12,39]. In bilayer graphene an additional degeneracy of the lowest LLL also occurs, because of zero energy of oscillatory Landau states with indices $n = 0$ and $n = 1$, due to double annihilation oscillatory operator in local Hamiltonian resulted from interlayer electron hopping in pseudo-relativistic bilayer graphene [39]. Despite different LL structure in graphene (in monolayer [12] with spectrum $\sim \sqrt{n}\frac{2\hbar v_F}{l_B}$ and in bilayer [39] $\sim \sqrt{n(n-1)}\hbar\omega_B$, $l_B = \sqrt{\frac{h}{eB}}$—the magnetic length, v_F—the Fermi velocity, $\omega_B = \frac{eB}{m}$—the cyclotron frequency; in comparison in GaAs 2DEG $\sim (n+\frac{1}{2})\hbar\omega_B$), all LL subbands in all materials have the same massive degeneracy N_0 resulted from the same ladder operators related to in-plane components of kinematic momentum at vertical magnetic field presence.

The fractional hierarchy of fillings for correlated states in all Hall materials results from the commensurability of cyclotron orbit size, $\frac{\Phi}{B}$ (here Φ is the magnetic field flux quantum) with interparticle spacing of 2D electrons rigidly fixed by the Coulomb repulsion [20,18,19]. The commensurability patterns define various homotopy phases governed by the value of the magnetic field flux quantum in a multiply connected configuration space [20],

$$\Phi = \begin{cases} \Phi_0 = \frac{h}{e} \ \textit{in simply connected space}, \\ \Phi_k = (2k+1)\frac{h}{e} \ \textit{in multiply connected space}. \end{cases} \quad (1)$$

In the above formula k enumerates the number of loops in braids linking electrons on the plane homogeneously distributed with separation fixed by the Coulomb repulsion, this separation between electrons on the positive uniform jellium is given by $\frac{S}{N}$. For sufficiently strong magnetic field the ordinary single-loop cyclotron orbits, $\frac{\Phi_0}{B}$, building braids with $k = 0$ are too-short and cannot match neighboring electrons $\left(\frac{\Phi_0}{B} < \frac{S}{N}\right)$. The quantum statistics and Hall correlations (identified by cyclotron braid subgroup generators and their unitary representations [21,20]) must be thus defined by larger cyclotron orbits $\frac{\Phi_k}{B}$ and related k-loop braids, large enough to match closest neighbors [20]. Various commensurability