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Market share dynamics using Lotka–Volterra models

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ABSTRACT

Although competition in the marketplace is inherently dynamic and firms change their competitive behavior over time, firms' competitive struggle is generally described using *autonomous* Lotka–Volterra (LV) models. A great limitation of autonomous LV systems is that the interaction coefficients are constant, and hence firms are assumed to have constant competitive strategies. Also, the solutions of LV models are generally unknown. To address these shortcomings, we introduce a class of *integrable* nonautonomous LV models. Our LV models present some relevant advantages. First, the analytical solutions of this system are known, therefore we no longer need to fit the LV coefficients. Second, the analytical solutions only depend on the *utility functions* of the competing firms. Third, our model has a strong connection with the logit model. As mainstream economics extensively use the logit model to describe market demand, our approach has solid economic foundations. In the second part of the article, we test the performance of our approach by studying two cases in competition economics. We find that our model has a better ability to describe and forecast market evolution than the LV *autonomous* models proposed in the literature.

1. Introduction

Although for many decades economists have considered market structure as one of the main determinants of firms' behavior (Bain, 1951), recent studies have shown that a narrow focus on the market shares of firms operating in a market can be misguiding (Baumol et al., 1982; Baumol, 1982). External forces and exogenous shocks (e.g. technological innovations and new regulations) can strongly influence the functioning of a market, and they often induce firms to change their competitive strategies (Modis, 1999, 2011). Therefore, the kind of competitive interaction among firms might change over time in response to technological innovations and exogenous shocks (Modis, 1999, 2011). However, abandoning the study of market shares altogether would not be advisable. Albeit imperfect, market shares are a reasonable proxy for market power and are often readily available. Instead, the lesson to be drawn is twofold. On the one hand, competition cannot be reduced to a static concept (Schumpeter, 1942; Aghion and Howitt, 1992). On the other hand, exogenous factors should be taken into account when explaining market dynamics. Despite the many attempts to use competition Lotka–Volterra models (LV) to describe market dynamics, very often these two factors have been overlooked.

Starting from the pioneering works (Lotka, 1925; Volterra, 1926), LV models have already been used for *modeling market competition dynamics* (Modis, 1999, 2011; Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López and Sanjulan, 2001; Michalakelis et al., 2011, 2012; Kloppers and Greeff, 2013; Lin, 2013; Kim et al., 2006; Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014; Kreng et al., 2012; Lee et al., 2005; Cerqueti et al., 2015). In these approaches, market evolution is estimated and forecasted considering market shares as species competing for a common source: the market potential. All these models refer to markets in which *the species competition roles are already established*; mostly predation (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López and Sanjulan, 2001; Michalakelis et al., 2012; Kloppers and Greeff, 2013; Duan et al., 2014; Cerqueti et al., 2015), but also commensalism (Lin, 2013; Kim et al., 2006), mutualism (Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014), neutralism (Wulf et al., 2011), and pure competition (Lakka et al., 2013; Cerqueti et al., 2015).

Although these models supply important contributions to the literature, they present some drawbacks:

(1) Each differential system is *autonomous*, i.e., the model equations contain only constant coefficients. Consequently, these models are based on the assumption that the economic factors affecting market shares' dynamics (e.g., marketing strategies and *species competition roles* (see Modis, 1999, 2011 and Table 1)) are constant over the time interval considered (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López

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Table 1
The competitive roles are deduced from the signs of d_{ij} and d_{ji}.

d_{ij}	d_{ji}	Type of interation
+	+	Pure competition
–	+	Predator–prey
–	–	Mutualism
–	0	Commensalism
+	0	Amensalism
0	0	Neutralism

and Sanjulan, 2001; Michalakelis et al., 2011, 2012; Kloppers and Greeff, 2013; Lin, 2013; Kim et al., 2006; Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014; Kreng et al., 2012; Lee et al., 2005; Cerqueti et al., 2015).

(2) Often the models are not adequately connected to the economic theory (López and Sanjulan, 2001). We mainly refer to the concepts of maximum market potential and the logistic market growth rates. Also, the models generally do not allow consumers to purchase a different kind of product (outside option) (for all these aspects see, for instance, Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Michalakelis et al., 2012).

(3) Each differential system admits only numerical solutions, i.e. the market shares as functions of time (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López and Sanjulan, 2001; Michalakelis et al., 2011, 2012; Kloppers and Greeff, 2013; Lin, 2013; Kim et al., 2006; Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014; Kreng et al., 2012; Lee et al., 2005; Cerqueti et al., 2015). Hence, it is difficult to link them to well-established economic models.

(4) The effectiveness of the proposed models passes through the estimations of the model parameters determining the competition roles. However, in the best case these estimations are based on a few of available data, and are achieved by means of expensive numerical methods as genetic algorithms (see Michalakelis et al., 2011, 2012; Lakka et al., 2013), integral and log integral methods (see Kloppers and Greeff, 2013), multiple linear regression (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Kreng and Wang, 2009, 2011; Tseng et al., 2014; Lin, 2013; Kim et al., 2006; Tsai and Li, 2009; Wulf et al., 2011; Duan et al., 2014; Lee et al., 2005), and so on

In this paper, we attempt to overcome all these problems. First, we render justice to the dynamic nature of competition by allowing firms to change their competitive behavior over time. In mathematical terms, we introduce a class of integrable nonautonomous LV systems describing the competition among N firms in a simulated market. This allows us to consider growth rates and interaction coefficients as dependent on time. Second, the LV model adopted in this paper has a strong connection with the *logit model* (McFadden, 1973; Budzinski and Ruhmer, 2010). As mainstream economics extensively use the logit model to describe the market demand (Nevo, 2000, 2001; Nevo and Rossi, 2008), our approach has solid economic foundations. Moreover, we include in our model an "outside option" to account for exogenous factors. In our framework, the outside option can be represented either by producers of a substitute to the examined product or by other producers of the same product. In the former case, we have a pure outside good. In the latter case, we have a "spurious" outside option synthesizing the information about the "fringe firms". We remark that our model allows us to include in the analysis every possible kind of exogenous shock. In fact, the effects of any shock can ultimately be reduced to two categories:

- A shock might induce firms to modify their competition strategies. This effect is captured by the dynamic nature of the growth rates and the interaction coefficients. Obviously, the traditional models with constant coefficients cannot take into account these effects.
- A shock might change the relative size of the market as a whole. In other words, the size of the relevant market in which firms compete usually changes over time. This effect is captured by the outside option. The greater the outside option is the smaller the market in which firms compete will be.
- Third, the analytical solutions of the LV system proposed are known, therefore we no longer need to fit LV coefficients. In turn, this allows us to address also the fourth problem. Because the analytical solutions only depend on the utility functions, we no longer need to fit LV coefficients.

The balance of this paper is as follows. In Section 2 we deal with the LV systems modeling market competition dynamics. We focus on model coefficients affecting the market growth. More precisely, we concentrate on product attractiveness, the maximum market potential, and the nature of the interaction (Modis, 1999, 2011).

In Section 3, we introduce a class of integrable nonautonomous LV systems describing the competition among N firms in a simulated market. Moreover, we study the analytic solutions of this class of models with respect to the *principle of competitive exclusion*. In Section 4 we illustrate a method for estimating the unknown model functions involved in the LV system. To this end, we analyze two sets of historical data representing the market shares of N competitors in two different markets (Michalakelis et al., 2011; Konishi and Yurtseven, 2014). In addition, we estimate the demand functions and, using the software *Mathematica®*, we compare the estimated market shares with the available historical data by means of the Mean Squared Error (*MSE*), the Mean Absolute Percentage Error (*MAPE*), and the Fractional Standard Deviation (*FS*). Section 5 shortly summarizes the main findings of the paper.

2. Lotka–Volterra models and market structure

A general competition Lotka–Volterra system of $N-$species in a established niche is expressed by the following ordinary differential equations.

$$\dot{x}_i(t) = \underbrace{a_i x_i(t) - b_i x_i(t)^2}_{\text{logistic growth}} - \underbrace{\sum_{j=1, j\neq i}^{N} c_{ij} x_i(t) x_j(t)}_{\text{interaction with competitors}}, \quad i = 1, \cdots, N, \tag{1}$$

where $x_i(t) \geq 0$ represents the population size of the $i-$th species at time t, the coefficients a_i (*growth rates*), b_i (*intraspecific competition*) and c_{ij} (*interaction coefficients*) are generally assumed to be constant, and $\dot{x}_i(t) = dx_i(t)/dt$.

Similarly, Eq. (1) describes the competition between N firms in a dynamic oligopoly market. The evolution of the *market shares* $x_i(t)$ of the $i-$th firm is determined by two factors: the logistic parameters a_i (intrinsic market growth rate) and b_i (intraspecific competition rate), and the competition rate c_{ij} between the $i-$th and $j-$th firm.

Simple manipulations allow us to reduce Eq. (1) to the following form.

$$\dot{x}_i(t) = \underbrace{a_i x_i(t)\left[1 - \frac{x_i(t)}{k_i}\right]}_{\text{logistic growth}} - \underbrace{\sum_{j=1, j\neq i}^{N} c_{ij} x_i(t) x_j(t)}_{\text{interaction with competitors}} \quad , \quad i = 1, \cdots, N, \tag{2}$$

where $k_i = a_i/b_i$ is the well known *carrying capacity*.

In the economic framework, the parameter $k_i = 1$ because it is the maximum potential of the market, i.e., the maximum capacity related to the saturation value of each market share $x_i(t), i = 1, \ldots N$.