Market share dynamics using Lotka–Volterra models

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A B S T R A C T
Although competition in the marketplace is inherently dynamic and firms change their competitive behavior over time, firms' competitive struggle is generally described using autonomous Lotka–Volterra (LV) models. A great limitation of autonomous LV systems is that the interaction coefficients are constant, and hence firms are assumed to have constant competitive strategies. Also, the solutions of LV models are generally unknown. To address these shortcomings, we introduce a class of integrable nonautonomous LV models. Our LV models present some relevant advantages. First, the analytical solutions of this system are known, therefore we no longer need to fit the LV coefficients. Second, the analytical solutions only depend on the utility functions of the competing firms. Third, our model has a strong connection with the logit model. As mainstream economics extensively use the logit model to describe market demand, our approach has solid economic foundations. In the second part of the article, we test the performance of our approach by studying two cases in competition economics. We find that our model has a better ability to describe and forecast market evolution than the LV autonomous models proposed in the literature.

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1. Introduction

Although for many decades economists have considered market structure as one of the main determinants of firms' behavior (Bain, 1951), recent studies have shown that a narrow focus on the market shares of firms operating in a market can be misleading (Baumol et al., 1982; Baumol, 1982). External forces and exogenous shocks (e.g., technological innovations and new regulations) can strongly influence the functioning of a market, and they often induce firms to change their competitive strategies (Modis, 1999, 2011). Therefore, the kind of competitive interaction among firms might change over time in response to technological innovations and exogenous shocks (Modis, 1999, 2011). However, abandoning the study of market shares altogether would not be advisable. Albeit imperfect, market shares are a reasonable proxy for market power and are often readily available. Instead, the lesson to be drawn is twofold. On the one hand, competition cannot be reduced to a static concept (Schumpeter, 1942; Aghion and Howitt, 1992). On the other hand, exogenous factors should be taken into account when explaining market dynamics. Despite the many attempts to use competition Lotka–Volterra models (LV) to describe market dynamics, very often these two factors have been overlooked.

Starting from the pioneering works (Lotka, 1925; Volterra, 1926), LV models have already been used for modeling market competition dynamics (Modis, 1999, 2011; Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López and Sanjulan, 2001; Michalakelis et al., 2011, 2012; Kloppers and Greeff, 2013; Lin, 2013; Kim et al., 2006; Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014; Kreng et al., 2012; Lee et al., 2005; Cerqueti et al., 2015). In these approaches, market evolution is estimated and forecasted considering market shares as species competing for a common source: the market potential. All these models refer to markets in which the species competition roles are already established; mostly predation (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López and Sanjulan, 2001; Michalakelis et al., 2012; Kloppers and Greeff, 2013; Duan et al., 2014; Cerqueti et al., 2015), but also commensalism (Lin, 2013; Kim et al., 2006), mutualism (Tsai and Li, 2009; Wulf et al., 2011; Lakka et al., 2013; Duan et al., 2014), neutralism (Wulf et al., 2011), and pure competition (Lakka et al., 2013; Cerqueti et al., 2015). Although these models supply important contributions to the literature, they present some drawbacks:

1. Each differential system is autonomous, i.e., the model equations contain only constant coefficients. Consequently, these models are based on the assumption that the economic factors affecting market shares’ dynamics (e.g., marketing strategies and species competition roles (see Modis, 1999, 2011 and Table 1)) are constant over the time interval considered (Chiang, 2012; Miranda and Lima, 2013; Chiang and Wong, 2011; Morris and Pratt, 2003; Kreng and Wang, 2009, 2011; Tseng et al., 2014; López
In this paper, we attempt to overcome all these problems. First, we render justice to the dynamic nature of competition by allowing firms to change their competitive behavior over time. In mathematical terms, we introduce a class of integrable nonautonomous LV systems to change their competitive behavior over time. In mathematical rendering justice to the dynamic nature of competition by allowing the information about the producers of the same product. In the former case, we have a pure outside option synthesizing. We remark that our model allows us to include in the analysis every possible kind of exogenous shocks. A shock might induce firms to modify their competition strategies. This effect is captured by the dynamic nature of the growth rates and the interaction coefficients. Obviously, the traditional models with constant coefficients cannot take into account these effects.

\begin{equation}
\dot{x}_i(t) = a_i x_i(t) - b_i x_i(t)^2/\sum_{j=1}^{N} c_{ij}(t)x_j(t), \quad i = 1, \ldots, N, \tag{1}
\end{equation}

where $x_i(t) \geq 0$ represents the population size of the $i$-th species at time $t$, the coefficients $a_i$ (growth rates), $b_i$ (intraspecific competition) and $c_{ij}$ (interaction coefficients) are generally assumed to be constant, and $x_i(t) = dx_i(t)/dt$.

Similarly, Eq. (1) describes the competition between $N$ firms in a dynamic oligopoly market. The evolution of the market shares $x_i(t)$ of the $i$-th firm is determined by two factors: the logistic parameters $a_i$ (intrinsic market growth rate) and $b_i$ (intraspecific competition rate), and the competition rate $c_{ij}$ between the $i$-th and $j$-th firm. Simple manipulations allow us to reduce Eq. (1) to the following form.

\begin{equation}
\dot{x}_i(t) = a_i x_i(t) \left[1 - \frac{x_i(t)}{K_i}\right] - \sum_{j=1}^{N} c_{ij}(t)x_j(t), \quad i = 1, \ldots, N, \tag{2}
\end{equation}

where $K_i = a_i/b_i$ is the well known carrying capacity.

In the economic framework, the parameter $K_i = 1$ because it is the maximum potential of the market, i.e., the maximum capacity related to the saturation value of each market share $x_i(t), i = 1, \ldots, N$. The balance of this paper is as follows. In Section 2 we deal with the LV systems modeling market competition dynamics. We focus on model coefficients affecting the market growth. More precisely, we concentrate on product attractiveness, the maximum market potential, and the nature of the interaction (Modis, 1999, 2011).

In Section 3, we introduce a class of integrable nonautonomous LV systems describing the competition among $N$ firms in a simulated market. Moreover, we study the analytic solutions of this class of models with respect to the principle of competitive exclusion. In Section 4 we illustrate a method for estimating the unknown model functions involved in the LV system. To this end, we analyze two sets of historical data representing the market shares of $N$ competitors in two different markets (Michalakelis et al., 2011; Konishi and Yurtseven, 2014). In addition, we estimate the demand functions and, using the software Mathematica®, we compare the estimated market shares with the available historical data by means of the Mean Squared Error (MSE), the Mean Absolute Percentage Error (MAPE), and the Fractional Standard Deviation (FS).

Section 5 shortly summarizes the main findings of the paper.
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