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Modeling seasonal effects in the Bass Forecasting Diffusion Model

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ABSTRACT

The Bass Forecasting Diffusion Model is one of the most used models to forecast the sales of a new product. It is based on the idea that the probability of an initial sale is a function of the number of previous buyers. Almost all products exhibit seasonality in their sales patterns and these seasonal effects can be influential in forecasting the weekly/monthly/quarterly sales of a new product, which can also be relevant to making different decisions concerning production and advertising. The objective of this paper is to estimate these seasonal effects using a new family of distributions for circular random variables based on nonnegative trigonometric sums and to use this family of circular distributions to define a seasonal Bass model. Additionally, comparisons in terms of one-step-ahead forecasts between the Bass model and the proposed seasonal Bass model for products such as iPods, DVD players, and Wii Play video game are included.

1. Introduction

Almost all products exhibit seasonality in their sales patterns. For example, the sales of many products increase in December or the fourth quarter of the year due to the bonuses paid by employers to employees or due to religious traditions, such as Christmas. Other seasonal human activities, for example, the beginning of the school year, increase the sales of certain products related to these activities. Finally, weather is one of the main causes of observed seasonal effects on the sales of different products.

The identification and estimation of seasonal effects on the sales of a product are very important for marketing decisions, such as determining the best time to introduce a new product, when to increase spending on advertising, the rotation of products in a store, and the amount to produce, among others. For a previous analysis of seasonal effects in marketing problems, the interested reader can consult Radas and Shugan (1998).

The Bass Forecasting Diffusion Model (Bass, 1969) is one of the most frequently used models for forecasting the sales of a new product. The main idea behind the Bass model is that the sales of a product at a given point in time depend on the number of buyers since the introduction of the product. In this sense, the buyers of the new product are classified as innovators and imitators. Innovators are buyers who buy the product without considering the recommendation of previous buyers, only buying in response to external influences, such as advertising. The timing of an innovator's initial purchase is not influenced by the number of buyers who have already bought the product. Imitators are buyers who buy the product because they have been influenced by the number of previous buyers; i.e., they buy in response to internal influences, such as word of mouth. This model is determined by three parameters, m, p, and q where m is the total number of people who eventually buy the product, p is the coefficient of innovation, and q is the coefficient of imitation. We refer to the original Bass model as the classical Bass model. Later, Bass et al. (1994) extended the classical Bass model to include the effects of explanatory variables such as price and advertising, on the sales of a new product. For reviews of the classical Bass model, its modifications, and its applications, one can consult Rao (1985), Mahajan et al. (1990), Parker (1994), Meade and Islam (2006), Mahajan et al. (2000), Chandrasekaran and Tellis (2007) and Peres et al. (2010).

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The inclusion of seasonal effects in the Bass model is related to the problem of temporal aggregation, as discussed by Putsis (1996) (see also Non et al., 2003). Putsis criticizes the fact that the classical Bass model invariably used annual data, not taking into account the available quarterly or monthly data, with the exception of the paper by Tigert and Farivar (1981). Putsis recommends that studies should use higher-frequency data whenever possible because the models presented in the paper, adjusted with quarterly data, outperform those estimated with annual data. Unfortunately, Putsis (1996) analyzed monthly and quarterly data series that were seasonally adjusted using the census X-11 moving average method prior to estimating the parameters. In contrast to Putsis (1996), one of the main objectives of this paper is the estimation of seasonal effects. Rao (1985), when comparing the forecasting performance of different diffusion models, considers that an important goal for future research is a comparative study using more refined data sets with more closely spaced points because such reports would provide more opportunities to forecasts and more precise and useful conclusions about model performance in predicting the time to peak sales.

For certain products, the modeling of seasonal effects in diffusion models is very important; this is true, for example, among pharmaceutical products designed to cure seasonal diseases e.g., the flu diseases and electronic products with higher sales during the fourth quarter. From a statistical point of view, working with aggregated annual data when quarterly or monthly data are available is equivalent to throwing away important information from the time series data. For example, when making a forecast with only 2 years of data, in monthly terms, 24 data points are available, but in aggregated annual terms, just two data points are available, which is not enough to estimate the three parameters of the classical Bass model. As indicated by Radas (2001, 2005), due to increasing global competition and the shortening of product life cycles, managers are not allowed to wait for several yearly data points; thus, models that account for seasonal effects are very relevant in practice. Additionally, many authors have reported that the parameter estimates of the classical Bass model are highly sensitive to the number of observations (see Chandrasekaran and Tellis, 2007). Parker (1994) provides a warning about the problem in which the classical Bass model will inadvertently interpret early variations in adoption, such as those due to seasonality in monthly or quarterly sales data, as inflection points or peaks that can produce unreasonable parameter estimates and forecasts.

One of the main modifications of the classical Bass model has been the inclusion of marketing mix variables to allow the parameters of the Bass model to vary with time. The Generalized Bass Model (Bass et al., 1994) considers the use of price and advertising to modify the p and q parameters, while other authors have considered the modification of the market potential and/or the diffusion parameters through the use of variables such as price, advertising, detailing, distribution, and direct-to-consumer advertising, among others (see the review by Ruiz-Conde et al., 2006). In this sense, the proposed seasonal Bass model in this paper can be considered a modified Bass model in which the diffusion parameters p and q are constant but modified through the use of a positive function $x(t)$ during a full cycle to model the seasonal effects. However, additional variables, such as price or advertising, are not included in this model. Again, this is an advantage when dealing with short time series to make forecasts and variables such as price has not changed since the introduction of the new product. The classical Bass model is a particular case of the proposed seasonal Bass model. Putsis (1998) compared models with constant parameter values to those with parameters following a stationary or a nonstationary stochastic process. Recently, Peers et al. (2012) used seasonal dummies to modify a closed-form innovation model in discrete time, in particular the Bass model, by considering that the seasonal peaks are the effect of postponed and accelerated sales in previous and future months. By working in discrete time, this model becomes too complicated in the case of multiple seasonal peaks, it is not able to deal with data given by combining different time frequencies and, requires many data points to make the estimation. Also, it is difficult to use this model in analogy forecasting. Contrary to Peers et al. (2012), the proposed model in this paper is derived in continuous time by the use of circular distributions based on nonnegative trigonometric sums (see Fernández-Durán, 2009). By considering continuous circular distributions, many of the disadvantages of the Peers et al. model are avoided and the proposed methodology can be applied to sales data observed in any frequency or even in different frequencies. The estimated circular distribution can be used in analogy forecasting for similar products to be launched in the future.

From meta-analysis studies of the classical Bass model (see the references in Chandrasekaran and Tellis, 2007), one finds that the mean value of the coefficient of innovation, p, lies between 0.0007 and 0.03, while the coefficient of imitation, q, lies between 0.38 and 0.53. Sultan et al. (1990) analyzed 213 applications of diffusion models and found that p is equal to an average of 0.03, and q is equal to an average of 0.38.

To address seasonality in monthly and quarterly sales data, it is common practice to use seasonally adjusted time series. The seasonal adjustments are performed using statistical methods, such as the X11 (see Ladiray and Quenneville, 2001), X12-ARIMA (see Findley et al., 1998), or TRAMO-SEATS (TRAMO: Time series Regression with ARIMA noise and SEATS: Signal Extraction in ARIMA Time Series, see Gómez and Maravall, 1998) procedures. Once the sales series has been adjusted for seasonality, then the classical Bass model is applied to the seasonally adjusted sales series to estimate the parameters (p and q) of the model (see Putsis, 1996). These parameter estimates are then used to forecast future sales. In this paper, the proposed Seasonal Bass model based on NNTS circular distributions is compared with models using seasonally adjusted data obtained by the X12-ARIMA and TRAMO-SEATS procedures.

Statistical procedures for seasonal adjustment consider that a time series, Y_t for $t = 1, \ldots, T$, can be decomposed into

$$Y_t = T_t + S_t + I_t \tag{1}$$

for an additive decomposition, or

$$Y_t = T_t S_t I_t \tag{2}$$

for a multiplicative decomposition, where T_t is the trend-cycle, S_t is the seasonal component, and I_t is the irregular