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An efficient decoupling dynamic algorithm for coupled multi-spring-systems

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ABSTRACT

In this paper, a novel explicit method for decoupling coupled multi-spring systems in the structural dynamic analysis is proposed by ensuring the second-order accuracy in the low-frequency domain. Two parameters μ and β are introduced in this method to flexibly adjust the stability and accuracy properties and suppress the high-frequency spurious vibrations in the solution. Firstly, the standard formulations of this method are derived and its stability and accuracy are analyzed through comparisons with other available state-of-the-art explicit methods in the literature. Then, the performance of this proposed method is specifically evaluated in terms of accuracy, dissipation in the high-frequency domain and computational efficiency, and effectiveness in dealing with nonlinearity through three examples. Finally, a vertical coupled multi-spring system, a train-rail-sleeper-ballast-bridge system, is employed to demonstrate the efficiency and decoupling properties of the proposed method. The presented attractive performance of this method illustrates its advantages in engineering applications.

1. Introduction

Direct time integration method is widely employed in solving structural dynamic problems and it consists of two categories: explicit and implicit methods [1,2]. Compared with the implicit methods with the property of unconditional stability, most of the explicit methods can only be designed to have conditional stability, except for a very few explicit methods [3–7]. Among these few explicit methods with the unconditional stability property, their parameters to be solved involve each frequency and viscous damping of the system, which results in low computational efficiency. Meanwhile, for the explicit method with conditional stability, the complicated and time-consuming factorization of the 'effective stiffness matrix' is not required in the solution [8–10], which largely improves the computational efficiency. However, for certain engineering problems with high-frequency vibrations, e.g., the crash simulation and the wheel-track coupling vibration analysis, a desirable small time step size is required in order to capture the responses of high-frequency vibrations. In this case, the efficiency and accuracy of the method, rather than the stability, are the critical factors for the selection of the integration method [11]. Therefore, explicit methods with capabilities for solving problems with high-frequency vibrations are often attractive.

The explicit method can be divided into two categories: single sub-step and multi sub-step methods. Some higher-order accuracy explicit methods with multi sub-step are summarized in Refs. [12–14], e.g., the fourth-order Runge-Kutta's method [15], Kujawski and Gallagher's method [16], and Kinmark and Gray's method [17]. For these methods, high derivative terms, e.g., the time derivative of acceleration, or more than one function, e.g. calculation of the internal forces, are involved, which requires high computational cost and large storage of vectors. Comparative studies [18] shows while a high accuracy can be obtained in the multi sub-step methods, the best compromise between computation time cost and data storage is reported in the single sub-step methods compared with the multi sub-step methods. Therefore, the multi sub-step methods are less efficient in performance than that for the traditional central difference method (CDM). Moreover, most of these methods usually have a lower performance on stability than the CDM. The comparison between the Runge-Kutta method and the CDM also proves that the superiority of the CDM, in terms of the computational cost, is established in the numerical experiments [14,19]. This motivates and enables the authors to conceive an explicit approach to be built upon the single sub-step method.

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In the practical engineering, while diagonalization of the mass matrix can be usually observed, diagonalization of the damping matrix poses high challenges to the practitioners. Therefore, for the CDM, although with the above discussed advantages, the time-consuming factorization of an effective damping matrix is inevitably required to analyze dynamic problems with off-diagonal mass and damping matrices. Furthermore, the CDM is a non-dissipation method. For a problem with high-frequency vibrations, its solution accuracy can be severely ruined by the numerical dispersion and dissipation errors [20,21]. Note that the dispersion and dissipation are resulted from the discretization of the spatial and time integration domain.

To suppress the high-frequency spurious vibrations, certain parameters have been introduced into explicit methods e.g., the explicit Newmark method [22], the Hulbert-Chung (HC) method [23], Chung and Lee [24], the Tchamwa-Wielgosz method (TW) [25,26] and Zhai method [27]. Noh and Bathe [28] shows that while the Newmark and Zhai methods feature the dissipation property for the high-frequency vibrations, both methods are only of the first-order accuracy and therefore, their solution accuracy decreases in the low-frequency domain. Comparative studies [18,28,29] shows the TW method is efficient in filtering the high-frequency oscillations and it is more dissipative than the HC method. While the accuracy of the TW method remains within the tolerance limits, it is also of the first-order, same as the Zhai method. As such, based on the review of typical state-of-the-art methods, e.g., the TW method with single sub-step and the Noh-Bathe method (NB) with two sub-steps, proposing a favorable explicit method with one sub-step and high order accuracy is another motivation of the current study.

To this end, a novel explicit method with single sub-step is proposed by attaining the second-order accuracy for the solution in the low-frequency domain. Two parameters μ and β are introduced to adjust the stability and accuracy of the proposed method and suppress the high-frequency spurious vibrations. Several state-of-the-art methods with certain high accuracy, the CDM, TW, the implicit trapezoidal rule (TR), and the NB with two sub-steps, are employed to comparatively evaluate the performance of the proposed method. More specifically, we first derive the standard formulations of the proposed method and analyze its stability and accuracy properties. Then, a low-frequency harmonic vibration example is adopted to evaluate the accuracy of the proposed method. In addition, a Howe truss with the high-frequency vibrations is analyzed to investigate the dissipation property in the high-frequency domain, as well as the computational efficiency of this proposed method. Furthermore, a spring-pendulum problem is considered to demonstrate the effectiveness of the proposed method in terms of nonlinearity. Finally, a vertical coupled multi-spring system, a train-rail-sleeper-ballast-bridge system, is employed to demonstrate the efficiency and decoupling properties of the proposed method.

2. Proposition and evaluation for the method

2.1. Standard formulations

The governing equation of structural dynamic vibrations with initial conditions can be expressed as:

$$\begin{aligned} &M\ddot{U} + C\dot{U} + KU = F \\ &U(0) = U_0 \; \dot{U}(0) = \dot{U}_0 \end{aligned} \tag{1}$$

where M, K and C are the mass, stiffness and damping matrices, respectively, F is an external excitation, U is the displacement vector, the superposed dot denotes a time derivative, and U_0 and $\dot{U}_0$ are the initial nodal point displacements and velocities, respectively.

To solve Eq. (1) with a desired accuracy, a general explicit three-step scheme (involving three time steps, $t+\Delta t$, t, and $t-\Delta t$) is proposed and expressed in Eqs. (2) and (3), where the displacement and velocity at time $t+\Delta t$ involve only the results at the previous time steps $t-\Delta t$ and t.

$$\begin{aligned} {}^{t+\Delta t}U = a_1{}^{t}U - a_2{}^{t-\Delta t}U + \left(a_3{}^{t}\dot{U} - a_4{}^{t-\Delta t}\dot{U}\right)\Delta t \\ + \left(a_5{}^{t}\ddot{U} - a_6{}^{t-\Delta t}\ddot{U}\right)\Delta t^2 \end{aligned} \tag{2}$$

$$\begin{aligned} {}^{t+\Delta t}\dot{U} = b_1({}^{t}U - {}^{t-\Delta t}U)/\Delta t + \left(b_2{}^{t}\dot{U} - b_3{}^{t-\Delta t}\dot{U}\right) \\ + \left(b_4{}^{t}\ddot{U} - b_5{}^{t-\Delta t}\ddot{U}\right)\Delta t \end{aligned} \tag{3}$$

where $a_1 \sim a_6$ and $b_1 \sim b_5$ are the coefficients to be solved or determined to develop a specific numerical scheme with the desired accuracy, and the superscripts, e.g., $t-\Delta t$, at the displacement U and velocity $\dot{U}$ denote specified calculation time steps for the variables. All items related to $t-\Delta t$ on the right side of Eqs. (2) and (3) are expanded as the Taylor series at the time t. By further manipulating the expanded Taylor terms, the requirements in Eq. (4) should be satisfied in order to ensure the desired second-order accuracy for the displacement and velocity in, respectively, Eqs. (2) and (3):

$$\begin{aligned} &a_1 - a_2 = 1 \\ &a_2 + a_3 - a_4 = 1 \\ &-a_2 + 2a_4 + 2a_5 - 2a_6 = 1 \\ &b_1 + b_2 - b_3 = 1 \\ &-b_1 + 2b_3 + 2b_4 - 2b_5 = 2 \\ &-b_1 + 3b_3 - 6b_5 = -3 \end{aligned} \tag{4}$$

Then, using the Eqs. (1)-(3) [23,24], the following difference equation can be derived:

$$6^{t+\Delta t}U + 3^{t}UB_1 - {}^{t-\Delta t}UB_2 + {}^{t-2\Delta t}UB_3 + {}^{t-3\Delta t}UB_4 = 0 \tag{5}$$

where $B_1 \sim B_6$ are written as:

$$\begin{aligned} B_1 &= \left(-4 - 2a_2 + 2b_1 - 2b_3 - (-1 - a_2 + 2a_4 - 2a_6)\Omega^2\right) \\ B_2 &= \begin{pmatrix} -6 - 12a_2 + 12b_1 + 6a_4b_1 - 6(2+a_2)b_3 \\ -\Omega^2\begin{pmatrix} 6 - 12a_2 + 15a_4 - 12a_6 + 5b_1 + a_2b_1 \\ -4a_4b_1 + 6a_6b_1 - 6b_3 + 3a_4b_3 - 6a_6b_3 \end{pmatrix} \end{pmatrix} \\ B_3 &= \begin{pmatrix} -6a_2 + 6b_1 + 12a_4b_1 - 6(1+2a_2)b_3 \\ -\Omega^2\begin{pmatrix} 3 - 3a_2 + 12a_4 - 6a_6 - b_1 + a_2b_1 \\ +a_4b_1 + 6a_6b_1 - 6a_2b_3 + 6a_4b_3 - 12a_6b_3 \end{pmatrix} \end{pmatrix} \\ B_4 &= \left(6(a_4b_1 - a_2b_3) + \Omega^2(a_4b_1 + 6a_6b_3 - 3a_4 - 3a_4b_3)\right) \end{aligned} \tag{6}$$

where $\Omega = \omega\Delta t$, and ω denotes the natural frequency. Here, in order to simplify the calculation of Eq. (5), it is assumed that $B_4 = 0$, which leads to:

$$\begin{aligned} &a_4b_1 = a_2b_3 \\ &a_4b_1 + 6a_6b_3 - 3a_4 - 3a_4b_3 = 0 \end{aligned} \tag{7}$$

By designating $\mu = a_6$, $\beta = b_3$ and $a_4 = 1/4$ (see discussions in Section 2.2), the coefficients in Eqs. (2) and (3) can be solved based on Eqs. (4) and (7) and they are expressed as follows:

$$\begin{aligned} &a_1 = 3/(4\beta) - 6\mu + 7/4 \\ &a_2 = 3/(4\beta) - 6\mu + 3/4 \\ &a_3 = 6\mu - 3/(4\beta) + 1/2 \\ &a_4 = 1/4 \\ &a_5 = 3/(8\beta) - 2\mu + 5/8 \\ &a_6 = \mu \end{aligned} \tag{8}$$

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