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On the application of curve reparameterization in isogeometric vibration analysis of free-from curved beams

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ABSTRACT

In the present paper, an innovative approach based on a curve reparameterization technique is developed to solve the natural frequency problem of free-form Euler–Bernoulli curved beams using isogeometric analysis (IGA). The method is beneficial in modifying the probably inappropriate initial parameterization of input geometries. For this purpose, a transformation from the initial parametric domain to a new reparameterized one is adopted, creating a pseudo arc-length parameterization that is suitable for IGA. An auxiliary one-dimensional B-spline curve is utilized as the parametric transfer function (PTF). A combination of curve rectification and B-spline approximation techniques is implemented to construct the PTF. The number of elements in the proposed approach is based on the knot vector of the PTF that is not dependent on the knot vector of the input NURBS curve defining the geometry. The method is tested on different numerical examples that prove the validity and applicability of the formulations.

1. Introduction

Isogeometric analysis (IGA) has effectively proved itself as a paradigm of bridging the gap between computer-aided design (CAD) and finite element analysis (FEA) [1]. The main feature of the IGA approach is that the shape functions of the solution space in FEA are considered the same as the basis functions defining the CAD geometry. This method has been successfully implemented to a wide range of engineering problems such as structural mechanics [2–6], fluid mechanics [7–8], heat transfer [9–10], eigenvalue problems [11–12], micro/nano mechanics [13–14], and composites and functionally graded materials [15–17]. An overview and computer implementation aspects of IGA can be found in [18]. Recently, isogeometric analysis of curved beams has attracted many researchers. Bouclier et al. [19] investigated the use of higher-order NURBS curves to study the locking phenomenon in straight and curved Timoshenko beams. Isogeometric sizing and shape optimization of the curved beams are studied by Nagy et al. [20]. Bauer et al. [21] and Hosseini et al. [22], by presenting a nonlinear iterative solution, considered the nonlinear effect of large deformations in Euler–Bernoulli and Timoshenko curved beams, respectively. Some other works related to isogeometric analysis of curved beams and curved structures can be found in [23–30]. The IGA approach has been also employed to solve many dynamic problems. Reali [31] and Weeger et al. [32] investigated the vibration analysis of Euler–Bernoulli beams. Luu et al. [33] employed an IGA approach to solve the free vibration problem of curved beams with variable curvature. They used a Tschirnhausen cubic curved geometry to study the dynamic behavior of free-form beams. Some other researches concerning the application of IGA in vibration analysis of structures may be found in [34–37].

Parameterization is, in fact, a hidden concept behind the geometry. Although the shape of the curves and the geometric features such as curvature are not dependent on the parameterization, other features like, e.g., the Jacobian, rely on the curve's derivatives and are totally related to the parameterization. The parameterization issue is a topic of interest in many engineering applications especially in robotics [38–40] and machining processes [41–43]. Parameterization is an important topic in IGA as well [44–46] since changing the parameterization will change the discretization of the geometry and can cause mesh distortion. An extensive research in this area has been conducted by Xu et al. in 2D [45] and 3D [46] cases. In the mentioned research works, the optimization methods are employed to obtain an optimal parameterization of computational domains. While keeping constant the boundary curves/surfaces, the optimizer changes the position of middle control points to reach a better parameterization of computational domains.

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However, it should be pointed that this method is not applicable to planar free-form curves, because changing the position of middle control points would change the shape of the curves, violating the exact geometry philosophy of IGA. Kolman et al. [47] studied two parameterization schemes for straight lines: a nonlinear parameterization given by uniformly spaced control points and a linear parameterization obtained by Greville abscissae. Lipton et al. [48] investigated the influence of perturbing control points in one- and multi-dimensional settings and there are some other studies addressing the effects of parameterization [49–50].

The current work may be considered as a contribution to the concept of analysis-aware modeling [44,51–52], which emphasizes that the model parameters and properties should be selected to facilitate isogeometric analysis. It was in fact proven by Hosseini et al. [53] that reconstructing a curve from data points by a chord-length parameterization leads to superior results for IGA and the deficiencies of an equally-spaced parameterization were addressed. Although the chord-length approximation is very suitable for IGA when the geometry is given in terms of input data points, there are many practical cases where the input is instead directly given in terms a B-spline/NURBS definition of the geometry, for instance when a curve is imported from a CAD software. In these cases, the parameterization is predefined in the input spline functions and not guaranteed to be appropriate for IGA. Inappropriate parameterizations could be generally expected in CAD geometries since curves/surfaces are created considering geometric/aesthetic constraints. Such unsuitable parameterizations may be disregarded in CAD modeling without any significant effect on the geometry. However, high local nonlinearities in the parameterization may cause ill-conditioned stiffness and mass matrices, which might strongly affect the IGA results. Data sampling and curve regeneration methods for such cases may lead to inaccurate shapes violating the exact geometry advantage of the IGA approach. In addition, in those situations, standard mesh refinement techniques are not always helpful. In such occasions, reparameterization may be an applicable method to change the curve parameterization while keeping the geometry in its exact configuration [38,54]. Accordingly, a reparameterization strategy for modifying a probably inappropriate and highly nonlinear initial parameterization of input curved geometries is herein presented with the goal of solving the natural frequency problem of free-form curved beams with variable curvature. Differently from the reparameterization approach adopted by Xu et al. [55], which performs a Möbius transformation to construct analysis-suitable trivariate NURBS solids, the proposed methodology aims at creating a pseudo arc-length parameterization that is similar to a chord-length one in the case of curve approximation. The new pseudo arc-length parameterization guarantees a reduced mesh distortion and, generally, less computational effort is needed considering that integration with respect to the arc length is frequently seen in isogeometric analysis of curved beams. This can also resolve the ill-conditioning issue of the stiffness and mass matrices in the case of highly nonlinear input parameterization. In the proposed methodology, the adopted parametric transfer function (PTF) governing the approach consists of a B-spline curve constructed by a combination of curve rectification [56] and one-dimensional B-spline approximation techniques. It will be shown by means of some practical case studies that, while a classical IGA approach leads to accurate results for quasi-linear or weakly nonlinear input parameterization, this may not be the case for geometries with highly nonlinear parameterization or having relatively sharp corners, whereas the proposed reparameterization technique guarantees accurate results in all cases. We finally remark that the proposed strategy can be straightforwardly applied beyond beams, and may be therefore conveniently adopted to solve other engineering problems involving different governing equations, e.g., in fracture mechanics [57–58] and elastically supported beams [59].

The remainder of this article is organized as follows. Section 2 presents an introduction to B-splines and isogeometric approximation methods. In Section 3, the method of constructing the PTF that leads to a new pseudo arc-length parameterization is introduced, followed by the formulation of the isogeometric natural frequency problem for variable-curvature Euler–Bernoulli curved beam in Section 4. In Section 5, various numerical examples are given to show the validity and applicability of the proposed methodology. Finally, Section 6 draws some conclusions.

2. B-spline curves

B-splines, as a subset of non-uniform rational basis splines (NURBS), play an important role in the isogeometric analysis of curved beams since they serve both for representing the geometry and for expressing the displacement fields. B-splines are generally interpreted as unweighted or non-rational form of NURBS curves that can model a wide variety of free-form geometries. They are also well consistent with commercial CAD software and employed in different engineering problems (see, e.g., [60–62]).

2.1. B-spline curves: definition

A B-splines curve of degree p, is expressed as a piecewise continuous parametric function with $n+1$ control points P^i as follows [54]:

$$r(\xi) = \sum_{i=0}^{n} N_{i,p}(\xi) P^i \quad (0 \leqslant \xi \leqslant 1) \tag{1}$$

where $r(\xi) = [x(\xi), y(\xi)]$ is a vector-valued function whose components are represented separately as explicit functions of the parameter ξ over the knot vector Ξ with $n-p+1$ non-zero knot spans. In addition, $N_{i,p}(\xi)$ is the i -th basis function of degree p defined by the Cox–de Boor recursion formula [54]:

$$\Xi = \left[\underbrace{0, 0, \cdots, 0}_{p+1}, \bar{\xi}_{p+1}, \bar{\xi}_{p+2}, \cdots, \bar{\xi}_{n}, \underbrace{1, 1, \cdots, 1}_{p+1}\right] \tag{2}$$

$$\begin{aligned} N_{i,0}(\xi) &= \begin{cases} 1 & \bar{\xi}_i \leqslant \xi < \bar{\xi}_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ N_{i,p}(\xi) &= \frac{\xi - \bar{\xi}_i}{\bar{\xi}_{i+p} - \bar{\xi}_i} N_{i,p-1}(\xi) + \frac{\bar{\xi}_{i+p+1} - \xi}{\bar{\xi}_{i+p+1} - \bar{\xi}_{i+1}} N_{i+1,p-1}(\xi) \end{aligned} \tag{3}$$

For example, Fig. 1 depicts a cubic B-spline curve with seven control points and four equally spaced non-zero knot spans, i.e. $\Xi = [0, 0, 0, 0, 0.25, 0.5, 0.75, 1, 1, 1, 1]$. This curve can be thought of as the geometry of a planar curved beam. As indicated in the figure, the first and last control points are coincident with the starting and ending points on the curve. It is also clear that in spite of having equally spaced internal knots, the arc length of the

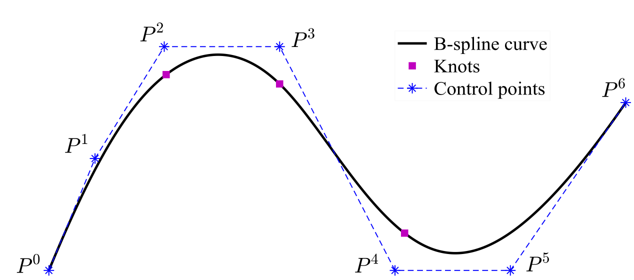

Fig. 1. A cubic B-spline curve with seven control points.

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