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Local absorbing boundary conditions to simulate wave propagation in unbounded viscoelastic domains

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ABSTRACT

In this paper, Local Absorbing Boundary Conditions are presented for wave propagation through the viscoelastic media. This method is an extension of Local Absorbing Boundary Condition (ABC) proposed by Lysmer and Kuhlemeyer (1969). The proposed method does not converge for Kelvin type of viscoelastic materials but converges for Maxwell type of viscoelastic material model. This method is verified with 1D and 2D finite element models using explicit solver and the accuracy is compared with standard solutions. This study concluded that better responses can be obtained for the viscoelastic wave propagation using present approach when compared with traditional Local Absorbing Boundary Conditions. This study also concluded that the proposed method is suitable for low to medium viscous damping materials. For high viscous materials, the proposed method can be applied to transient wave propagation problems.

1. Introduction

The effects of Structure-Soil-Structure-Interaction (SSSI) are being studied on the behavior of civil engineering structures for decades [1]. In order to solve the wave propagation problems using finite element analysis (FEA), the model has to be terminated at some finite location. This truncation of the model at the finite boundary will cause the reflection of radiating elastic waves. The reflected waves from the boundary will affect the solution and may lead to instabilities in the numerical analysis. Therefore, it is necessary to provide an artificial boundary condition that will transmit the outward propagating waves with minimum or negligible reflections. To address this problem, various kinds of analytical formulations have been developed, but they have their own limitations in avoiding reflections.

Absorbing Boundary Conditions (ABC) proposed by Lysmer and Kuhlemeyer [2] are the first local absorbing boundary conditions for elastic wave propagation. These boundary conditions are extensively used in commercial software, since they are very easy to implement, and have negligible computational cost. Engquist and Majda [3] and Mur [4] proposed higher order local approximate boundary conditions.

Perfectly Matched Layer (PML) was originally proposed by Berenger [5] is an artificial absorbing layer and has been widely used in recent year [6–10]. PML has been successfully implemented in time-domain for explicit dynamic solver by Basu in 2009 [11]. The basic idea of the PML is that the incident wave energy is absorbed inside the PML layers while matching impedance with non-PML layers. However, PML layers are derived based on elastic wave propagation. Also, the resulting finite element equations are very complex and require much computational time.

The Caughey Absorbing Layer Method (CALM) was proposed by Semblat (2010), is a simple and reliable alternative to the Perfectly Matched Layer (PML) like other absorbing layer methods [12–15], the CALM consists of defining an absorbing layer at the boundaries of the elastic medium under consideration. This absorbing layer is modelled with the same elastic properties as the interior medium, but the damping is added to attenuate all waves that leave the interior domain. This method also requires many absorbing layers to absorb the outward propagating waves and hence requires much computational time [15].

Apart from ABC and Absorbing Layers (PML, CALM and etc.,), techniques have been proposed to map the semi-infinite domain onto a finite domain using Infinite element [16–18]. The accuracy of the infinite elements depends upon the choice of the shape functions and the order of approximation. The dynamic Infinite elements [18] includes the effect of wave propagation into the

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unbounded domain by using frequency dependent mass and stiffness matrices. This method requires complex transformations of mass and stiffness matrices from the frequency domain to time domain.

It is well-known fact, almost all physical domains like soils exhibit some viscosity when the waves travel through these domains. It is also known that direct time integration is widely used in finite element solutions of structural dynamics and transient wave propagation problems. Viscous effects are usually considered in the form of Rayleigh damping for direct integral methods. As stiffness proportional damping is usually neglected for its significant influence on limit time increment, it is general practice to use mass proportional damping [19].

Local Absorbing Boundary Conditions are derived based on elastic wave propagation and produce reflections when used with viscoelastic material even when the wave impinges at a normal direction to the boundary. Nonlocal Absorbing Boundary conditions such as Caughey Absorbing Layer Method CALM, Artificial Layers by Increase in Damping (ALID) can be applied to viscoelastic wave propagation. However, these methods required many additional layers beyond the Area of Study (AoS) region to absorb the outward propagating waves efficiently Though, these methods effectively absorb the wave energy, if the successive layers are properly modelled, the number of degrees of freedom increases tremendously for three-dimensional wave propagation problems [15].

In this study, an attempt has been made to develop the local Absorbing Boundary Conditions for viscoelastic wave propagation by including viscosity in the form of mass proportional Rayleigh damping. The efficiency of the Viscoelastic Absorbing Boundary Conditions (VABC) has been verified by comparing the results with analytical models. The results show the better absorption of incoming waves from the viscoelastic domain and thus they can be used in SSSI problems which ensures more accuracy in the analysis.

2. Formulation of the method

The Absorbing Boundary Conditions correspond to a situation where the boundary is supported on infinitesimal dash-pots oriented normal and tangential to the boundary. The corresponding stress components are given by

$$\sigma = a\ \rho\ V_p \dot{u} \tag{1}$$

$$\tau = b\ \rho\ V_s \dot{v} \tag{2}$$

where σ and τ are the normal and shear stresses, $\dot{u}$ and $\dot{v}$ are the normal and tangential velocities respectively; ρ is the mass density; V_s and V_p are the velocities of S-waves and P-waves respectively; a and b are dimensionless parameters.

The equation of motion of the system under dynamic equilibrium is defined as

$$[M]\ddot{u} + [C]\dot{u} + [K]u = F \tag{3}$$

where $[M]$, $[C]$, and $[K]$ are the global mass, damping and stiffness matrices respectively. $\ddot{u}$, $\dot{u}$, u and F are the acceleration, velocity, displacement and external force vectors respectively. Damping matrix can be defined using Rayleigh damping coefficients as

$$[C] = \alpha[M] + \beta[K] \tag{4}$$

where α and β are mass and stiffness proportional damping coefficients. The equation of motion defined in Eq. (3) has the harmonic solution. The displacements, velocities, and accelerations can be expressed as

$$\begin{aligned} u_{(x,t)} &= \varphi e^{i(kx-\omega t)} \\ \dot{u}_{(x,t)} &= -i\omega\ \varphi e^{i(kx-\omega t)} \\ \ddot{u}_{(x,t)} &= -\omega^2\ \varphi e^{i(kx-\omega t)} \end{aligned} \tag{5}$$

where φ is amplitude, ω is angular frequency and k is wave number i.e. $k = \omega/V_p$. From Eqs. (3)–(5), the equation of motion can be written as

$$[\acute{M}]\ddot{u} + [\acute{K}]u = F \tag{6}$$

where $[\acute{M}] = [M](1 - \frac{\alpha}{i\omega})$ and $[\acute{K}] = [K](1 - i\omega\beta)$ are complex mass and stiffness matrices respectively.

Since the mass and stiffness are proportional to the density and Young's modulus respectively. The complex mass and complex Young's modulus can be calculated as

$$\begin{aligned} \acute{\rho} &= \rho\left(1 - \frac{\alpha}{i\omega}\right) \\ \acute{E} &= E(1 - i\omega\beta) \end{aligned} \tag{7}$$

Replacing Young's modulus and density in Eq. (1) with complex Young's modulus and complex density respectively

$$\sigma = a\ \rho\ V_p\sqrt{\left(1 - \frac{\alpha}{i\omega}\right)(1 - i\omega\beta)}\dot{u} \tag{8}$$

Expanding square root terms using Taylor series and ignoring higher order terms in the above equation

$$\sigma = a\ \rho\ V_p\left(1 - \frac{\alpha}{2i\omega} - \frac{i\omega\beta}{2} + \frac{\alpha\beta}{2}\right)\dot{u} \tag{9}$$

Back substituting Eq. (5) into the Eq. (9) yields

$$\sigma = a\ \rho\ V_p[0.5\beta\ \ddot{u} + (1 + 0.5\alpha\beta)\dot{u} + 0.5\alpha u] \tag{10}$$

Similarly, Eq. (2) can be modified by replacing the Young's modulus and density with the complex Young's modulus and complex density and by following the steps from Eqs. (8)–(10).

$$\tau = b\ \rho\ V_s[0.5\beta\ddot{v} + (1 + 0.5\alpha\beta)\dot{v} + 0.5\alpha v] \tag{11}$$

Eqs. (10) and (11) are the improved absorbing boundary conditions, which include the effect of the Rayleigh damping in the equation of motion. From Eqs. (8) and (9) it can be observed that stiffness proportional damping terms are not converging at higher frequencies, these equations are mainly limited to mass proportional damping cases.

If mass proportional only damping is applied, then Eqs. (10) and (11) can be rewritten as

$$\sigma = a\ \rho\ V_p\dot{u} + 0.5a\ \rho\ V_p\alpha u \tag{12}$$

$$\tau = b\ \rho\ V_s\dot{v} + 0.5b\ \rho\ V_s\alpha v \tag{13}$$

Absorbing forces in normal and tangential directions at the VABC can be calculated using $F_n = \sigma A$ and $F_t = \tau A$, where A is the associated area corresponding to respective dashpots. It can be observed from Eqs. (12) and (13) that the absorbing forces at the boundary include a dashpot with coefficient $a\rho V_p A$ and a spring with coefficient $0.5a\rho V_p \alpha A$.

The damping and spring coefficients are calculated only once at the begging of the solver and need not be updated during the analysis. Also, most of the existing Finite Element software packages allow defining dampers and springs. Therefore, these boundary conditions can be easily modelled without any additional implementation and the additional computation cost is negligible.

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