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# A multiscale force-based curved beam element for masonry arches

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This paper presents a Timoshenko beam finite element for nonlinear analysis of planar masonry arches. Considering small displacement and strain assumption, the element governing equations are defined according to a force-based formulation that adopts three different parametrizations of the axis planar curve, permitting the exact description of the element geometry for arbitrarily curved arches. Specific quadrature techniques are illustrated to perform numerical integration over the curved axis. A two-scale arch-to-beam homogenization procedure reproduces the nonlinear response of periodic masonry materials, where an equivalent Timoshenko straight beam describes the behavior of the reference Unit Cell made of a single linear elastic brick and a nonlinear mortar layer. Formation of the hinges characterizing the collapse mechanism of the arch is detected taking advantage of the quadrature rule along the axis and a fracture energy based regularization technique is employed to avoid damage localization.

The proposed curved beam model is implemented in a standard FE analysis code and is used to perform several numerical applications. After validating the proposed formulation through benchmarking tests under linear elastic material response, the numerical simulation of six experimental tests is shown, concerning masonry arches characterized by different shapes and undergoing in-plane bending. The numerical results are validated through experimental outcomes and other FE models.



## 1. Introduction

Until first decades of 1900, masonry has been the most used building technology in Europe for both civil buildings and cultural heritage constructions. However, due to low tensile strength of mortar and weak bond at interfaces with bricks and/or blocks, masonry structural elements exhibit a poor tensile/flexural resistance, that reduces the structure capacity and limits the design solutions [1]. For vertical dead loads, a widely adopted solution has long been the introduction of arches and vaults [2], as their particular geometry reduces presence of tensile stresses and increases carrying capacity of the structure (shape-based resistance). Arches are especially used in wall panels and for long span bridges, as their behavior under dead loads is easy to understand and predict. However, the response of curved elements under variable loads is complex and its simulation can be a hard task [3,4], particularly for horizontally seismic actions. Damage and micro-cracks usually develop along arch axis, even for small load levels, mainly due to debonding phenomena at interfaces and mortar failure. These influence the overall nonlinear response of the structure and determine collapse mechanisms. Hence, development of efficient numerical tools for nonlinear analysis of masonry arches still is a relevant issue for assessment of safety and strengthening of existing structures.

Many different models have been proposed to reproduce response of masonry structural elements and, in particular, masonry arches, commonly classified as mechanism methods (e.g. limit analysis), discrete element methods, discontinuous deformation analysis and finite element (FE) models [5–7]. The latter result the most adopted and flexible option, as these permit a complete representation of the nonlinear mechanisms with a reasonable demand in terms of computational costs. Micromechanical, macromechanical and multiscale approaches are three widely employed alternatives for FE analyses of masonry [8]. The first are based on distinct modeling of brick/blocks, mortar joints and/or interfaces; the second model masonry as an equivalent homogenized material and adopt phenomenological constitutive laws; the latter evaluate the response of a Unit Cell (UC) at a lower geometric scale and derive the response of the equivalent homogeneous medium at macro-scale through a homogenization process [9]. Multiscale models are a good compromise between detailed geometric/mechanical representation of the material and computational efforts.

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Focusing on masonry arches, beam FEs can be suitably used to model curved elements and define multiscale procedures, as these provide very accurate results compared to the limited computational burden required. Among different beam FE models, last decades have witnessed a large spread of force-based (FB) and mixed formulations for numerical analyses. As opposed to classic displacement-based (DB) models, where cross-section displacements are interpolated along the element axis by means of shape functions, FB beam formulations, also known as flexibility approaches, interpolate internal forces by means of functions satisfying element equilibrium in a strict sense. Hence, the selected interpolation functions are the exact solution to equilibrium equations regardless the element geometry and material constitutive law [10,11]. This is a consolidated procedure for straight beams and has shown superior performances with respect to DB models in reproducing the structural response [12], particularly under nonlinear material behavior [13,14]. In fact, accurate inelastic solutions can be obtained significantly reducing the number of elements used for the discretization. The FB beam formulation has also been proven to be free from shear locking, giving the exact analytical response for the elastic Timoshenko beam theory with a single FE per span [15,16]. A number of alternative strategies have been proposed to overcome the shear locking affecting the classic displacement-based FE formulations, such as selective reduced integration, use of quadratic polynomials for the transverse displacement and linear interpolation for the rotation, or interdependent interpolation of the transverse deflection and rotation derived on the basis of the exact solution for the Timoshenko beam theory [17]. Assumed strain methods have also been efficiently adopted, where the strains are independently interpolated and shear locking phenomena are avoided [18].

However, for both DB and FB formulations, few models consider the curved axis geometry, as the displacement and stress interpolation procedures are hard to define. The adoption of straight FEs combined with a mesh refinement is usually a preferred option to model the geometry of curved elements.

Besides the many proposals limited to linear elastic material assumptions and usually employed for large displacements and/or vibration analyses [19–24], the models in [25,26] are worth to be mentioned for nonlinear material modeling, yet belonging to DB formulations. For two-dimensional (2D) arches, Molari and Ubertini [27] have proposed a robust and efficient FB beam model, where the curved axis geometry is approximated with a cubic interpolation scheme. This approach allows the analysis of arbitrarily curved elements and, thus, results in a general formulation for the analysis of planar curved structural members. However, the study in [27] is limited to linear elastic material responses and does not investigate the model capabilities in the nonlinear material range. Moreover, the interpolation procedure for the axis approximation requires a pre-processing phase, where geometric parameters are determined for each structural members.

This work presents a general FB Timoshenko model for 2D curved beams under both linear elastic and nonlinear material responses, assuming small displacement and strain hypotheses. The element formulation considers the classic six displacement nodal degrees of freedom (DOF) and provides an exactly equilibrated representation of the cross-section stress resultants. In addition to the case of arbitrarily curved beams, this work considers two special cases for the axis geometry description, common in civil structures: constant curvature beams (circular arches) and parabolic arches. For both cases, the element governing equations can be easily derived relying on the exact representation of the axis geometry through a suitable parametrization of the 2D curve equation. This approach considerably simplifies model formulation and does not require any pre-processing procedure to describe the geometry of the structure. Moreover, specific quadrature techniques are proposed to efficiently evaluate integrals over the axis involved in element governing equations. These depend on the chosen geometric parametrization and strongly influence the computational costs.

The multiscale model proposed in [28] is introduced for the evaluation of the mechanical response of the general element cross-section. At the micro-scale level, a one-dimensional (1D) beam model describes the behavior of the UC and the homogenized macroscopic stress resultants at the curved beam quadrature point associated to the UC are determined via a semi-analytical approach. The bricks are considered as elastic, while a damage-friction constitutive law is used for the mortar.

In this model, the nonlinear hinges forming along the arch coincide with the zones where damage and plasticity localize and consequently their formation is monitored by following evolution of nonlinear mechanisms at the quadrature cross-sections. As usual when considering softening materials, damage localization requires the adoption of a regularization technique. The most common regularization approaches are the nonlocal integral procedures, the gradient methods [29], the fracture-energy based techniques [30] and the approaches proposed by Addessi and Ciampi [14] and Scott and Hamutçuoğlu [31] for FB beams. Nonlocal integral procedures and gradients methods are hard to introduce in the element formulation and increase computational burden, whereas generalization of the approach in [14] to curved beams is not straightforward. Hence, to prevent damage localization issues, a fracture energy regularization method is used in this work, where the evolutionary parameters of the damage model are calibrated at each cross-section considering the associated length defined for the quadrature rule.

The proposed curved beam model is implemented in a standard FE analysis code and used to perform various numerical applications. Two tests validate the model and the proposed quadrature techniques for linear elastic material response and investigate its advantages with respect to existing proposals. Nonlinear analyses on real masonry structures are also performed to study the capability of the model in detecting the nonlinear hinge formation and collapse mechanism of the members. The numerical results are validated through experimental outcomes and other proposed FE models.

## 2. FE model for 2D shear-deformable arches

This section describes the FB formulation of a 2D curved beam FE for the analysis of structural arches under nonlinear material response. The plane curve representing the element axis is assumed to have $C^0$ continuity and small displacement and strain assumptions hold. The FE model is based on the classic Timoshenko beam theory and results free from shear-locking problems.

The description focuses on the most suitable coordinate parametrization for the analytical representation of the axis geometry. First, the general case is described, where the 2D curve defining the beam axis is assumed to be known in a general parametric representation of the two coordinate functions. Then, the formulations for the circular and parabolic arches are presented. The derivation for the parabolic arch is introduced as an example procedure to obtain element governing equations, when the beam axis geometry is known in explicit form.

As usual in nonlinear beam FEs [14,31], the integrals over the axis involved in the element governing equations are evaluated through numerical integration procedures. Hence, the following study also describes the numerical approaches applied for the different beam formulations. These depend on the specific parametrization adopted to describe the axis geometry and permit

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