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Collapse displacements of masonry arch with geometrical uncertainties on spreading supports

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ABSTRACT

This work is aimed at evaluating the collapse displacement of masonry arch subjected to spreading supports. This is achieved through a general application of the virtual works principle. The problem is described in a finite displacements formulation and investigated with a probabilistic approach, also considering the effects of the geometrical uncertainties. This aspect is related to the imperfections of the voussoirs, which affect the structural shape. The comparison between the numerical and experimental results, derived both by the literature and laboratory tests, confirms that the geometrical irregularities can significantly affect the results obtained on the nominal structural geometry. Moreover, the disagreement observed in the experimental tests is explained.

1. Introduction

The masonry arch is one of the most commonly used structural components in the historical constructions. In the last centuries, the understanding of its behaviour has received a growing interest of architects, engineers and researchers, especially for the development of the scientific method. As for the more general cases of vaulted systems, the main function of a masonry arch is to bring the upper loads through specific ways of the structure to the ground, covering small or large spaces. The definition of the bearing capacity is a crucial task for the right dimensioning of an arch. In the case of restoration and/or retrofitting of existing buildings, bearing capacity is also fundamental for its check and validation. In the last decades, the scientific literature on this topic has considerably grown and the level of knowledge has significantly increased. In the second half of the XX century, a fundamental contribution was provided by Heyman [1,2], who used limit analysis for the study of masonry structures with an efficient approach for the rapid evaluation of the structural limit conditions. In this work, conceivable simplified hypotheses were assumed: no-tensile material, infinite compressive strength and no-sliding condition at failure between the voussoirs. The method is based on the well-known safe theorem, which states that "if a set of internal forces in a masonry structure can be found that equilibrate the external loads, and which lie everywhere within the masonry, then the structure is safe – safe in the sense that it cannot collapse under those loads" [3]. After Heyman's model, the upper bound and the lower bound methods or, alternatively, the limit equilibrium state analysis have been largely used. These methods were applied with several purposes, as the definition of the minimum thickness and/or the bearing capacity under vertical and lateral loads for different shapes of arches [4–11], the study of arches and vaults behaviour by using the thrust network analysis [12–15] or advanced numerical methods [16–19], the analysis of the strengthening effects through innovative materials [20–25] and many others.

During its life, a masonry arch has to withstand several threats that could significantly reduce its bearing capacity. This problem can be mainly related to two aspects: (i) structural damages of the arch (e.g. openings or slidings between the voussoirs due to load actions) and/or material degradation (reduction of the arch thickness or the strength of materials); (ii) springing settlements.

As far as it concerns the evaluation of structural and material degradation effects, in the last years several works have been focused on the assessment of the strength or stability reduction of a masonry arch due to its irregular geometry. The problem was investigated by modelling masonry arches taking into account the actual stones dimensions [26,27]. Elsewhere, parametric studies were applied to investigate the influence of localized damages [28,29] or probabilistic approaches were used for the estimation of uncertainties effects on the bearing structural capacity considering horizontal loads, both in static [30] and dynamic conditions [31]. These works emphasized that in the most cases the reduction of the collapse loads, with respect to the results obtained on the structures having nominal geometries, cannot be neglected.

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Regarding the study of the springing settlements effects, it can be stated that some aspects concerning the structural response of masonry arches – and more in general of masonry vaults – still present open problems. Differential settlements can be considered one of the main causes of collapse of vaulted structures [4], occurring for slow long-term deformations, for example due to static loads, or very quickly dynamic behaviour of the building, as in the case of earthquake actions. In a study concerning settled pushing structures, in particular arches and domes, Como [32] demonstrated "that, if the geometry changes are negligible, the structure will attain the minimum thrust state, saving its safety margin as in the perfect state". Ochsendorf [33] analysed the collapse conditions of the masonry arch on spreading supports in horizontal direction as a function of the geometrical parameters, namely the curvature radius, the thickness and the angle of embrace. Experimental results pointed out that the hinges may move with the increase of the settlements before reaching the collapse. Galassi et al. [34,35] studied the response of masonry structures to settlements considering rigid blocks connected by unilateral contact and frictional links, through a non-linear numerical procedure experimentally validated. Starting from the work of Ochsendorf, Coccia et al. [36] and Di Carlo et al. [37] developed an incremental procedure, based on the kinematic theorem applied to the deformed configuration. They aimed at attaining the collapse conditions of the masonry arch with horizontal spreading supports by varying the geometrical parameters and the number of voussoirs. Zang et al. [38] and Tubaldi et al. [39] analysed the masonry arch on spreading supports through a mesoscale modelling strategy, considering solid elements for bricks connected by interface elements for mortar joints. Constitutive models allow to consider the effects related to the possible presence of damages. Recently, Zampieri et al. analysed the effects of local pier scour in a multi-span masonry bridges [40] and the influence of no-horizontal springing supports of the masonry arch on the collapse mechanisms, with a numerical approach supported by experimental observations [41,42].

As pointed out by literature works, numerical simulations carried out on nominal geometry models seem to overestimate experimental results [4,33,36]. Starting from this point, this paper is aimed at investigating the role of geometrical irregularities, evaluated through a probabilistic approach, on the collapse conditions of a masonry arch subjected to spreading supports, which could be also related to abutments or piers deformations. In particular, the collapse conditions are studied through an incremental numerical procedure using the virtual works principle applied at the deformed configuration. For each deformed configuration, the limit equilibrium approach is used to assure the structural equilibrium and the strength condition defining the right hinges configurations. This condition occurs when the thrust line is contained inside the arch and passes through the hinge points.

Considering two experimental tests, in this work it is demonstrated for the first time that the reduction of the ultimate displacement observed at collapse, can be related to geometrical uncertainties, if compared with numerical simulations. This aspect leads to the opportunity of introducing safety factors in order to take into account such effects also in engineering practice [30].

2. Problem statement and numerical procedure

2.1. Basic hypotheses

Let us consider a circular masonry arch of radius r, thickness t and angle of embrace α made by n voussoirs under only its own weight in equilibrium state (Fig. 1). The generic ith voussoir is subjected to the vertical force

$$g_i = \gamma\, A_i d \tag{1}$$

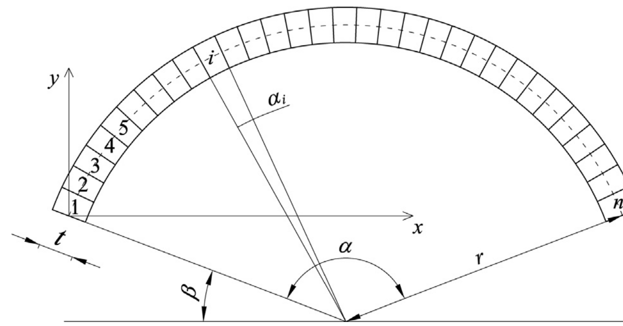

Fig. 1. Illustration of a masonry arch, divided in n voussoirs, with its geometrical parameters: radius r, thickness t and angle of embrace α.

where γ is the specific weight, A_i the area of the ith voussoir and d the constant out-of-plane depth. The arch is supposed to be fixed on a spreading support, in particular the left support without loss of generality (point $P_0(x_0, y_0)$ in Fig. 2), and the direction of the settlement δ_0 identified by the angle θ with respect to the horizontal. Given the geometrical parameters, the Cartesian coordinates of a generic point belonging to the arch can be indicated as a function of the radius r, thickness t and angle of embrace α. As an example, with reference to the Oxy system indicated in Fig. 1, the coordinates of the centre of mass of the ith voussoir are

$$x_i = r\cos\beta - r\cos\left(\beta + \frac{\alpha_i}{2} + (i-1)\alpha_i\right) \tag{2}$$

$$y_i = -r\sin\beta + r\sin\left(\beta + \frac{\alpha_i}{2} + (i-1)\alpha_i\right) \tag{3}$$

being $\beta = (\pi - \alpha)/2$ and $\alpha_i = \alpha/n$.

The passage form the initial unsettled configuration Ω^0 to an equilibrated settled configuration Ω^k induced by the spreading support is described by a kinematic mechanism consisting of a three-hinged rigid body chain.

The mechanism can be analysed with the well-known hypotheses proposed by Heyman [1]: (i) mechanism condition, (ii) resistance criterion and (iii) equilibrium condition. The first condition (i) requires that the mechanism is only of rotational type, so that no sliding can occur at each joint; the second (ii) considers a material with infinite compressive strength and no-tensile strength; the third (iii) corresponds to the individuation of a thrust line – equilibrated with the external loads – everywhere contained inside the arch parts profile and passing through the hinges. The ultimate state of equilibrium is reached by progressively increasing the value of the displacement δ_0 up to the loss of stability of the arch. This condition leads to the structural collapse with a mechanism which may involve either all the voussoirs, with a five-hinges symmetric mechanism, or a part of them, with the occurrence of an asymmetric configuration. In this case the collapse may develop starting from a four-hinges mechanism, or due to the alignment of the three hinges already present (three-hinges mechanism).

Let us consider the settlement δ_0 assigned in P_0 along θ direction and the resulting kinematically admissible displacement field $\delta(u, v)$ of the structure, with u and v as displacement components in x and y directions respectively. The equation of the virtual works – employed in the small displacement field – provided by the equilibrated settled configuration Ω^k and a virtual displacement field δ^{k*} having the same properties previously described (i.e. δ_0^{k*} defined in θ direction and δ^{k*} kinematically admissible) is

$$\langle g, \delta^{k*}\rangle + R_0^k \cdot \delta_0^{k*} = \langle \sigma^k, \varepsilon^{k*}\rangle \tag{4}$$

where σ^k and ε^{k*} are the stress and strain fields respectively, and R_0^k is the reaction force acting on P_0 along θ direction. In the Eq. (4), the