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Path following techniques for geometrically nonlinear structures based on Multi-point methods

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ABSTRACT

The Multi-point methods are efficient and accurate techniques for solving nonlinear equations. In this article, these methods are used to develop incremental/iterative techniques for nonlinear analysis of structures. The numerical results show that these methods only have the ability to converge to the equilibrium path before the first limit-point. To improve the performance of Multi-point methods, modified techniques which have the ability to fully trace the geometrically nonlinear response of structures are proposed in this paper. Four novel algorithms are presented which follow the defined constraint while solving the equilibrium equations using modified Multi-point methods. The Multi-point methods and the modified ones are comparatively investigated for the geometrically nonlinear analysis of structures in both continuum and discrete problems (dome and cylindrical shell). Selected examples represent a host of nonlinearities, including large fluctuations in stiffness, snap-back, and snap-through behaviors. The numerical results show that the modified methods have the ability to fully capture the geometrically nonlinear response of structures, including snap-back and snap-through behaviors.

1. Introduction

One of the common issues in structural and continuum mechanics are nonlinear problems. Many attempts have been made to develop effective and practical algorithms in nonlinear analysis of structures. Till date, there is no general method which can be capable of solving all nonlinear structural problems [1]. Depending on the characteristics of the nonlinear problems, one method may be preferred over another [2]. In nonlinear analysis, the structures' behaviors are modeled as a system of nonlinear algebraic equations, and the solutions of the nonlinear system of equations indicate the responses of the structures and embody their behaviors. The Iterative numerical procedures are the methods of choice for finding the solutions of these nonlinear systems. The iterative methods for solving nonlinear equations include two categories of one-point and multi-point methods. The one-point methods only use one evaluated point for approximating the next point. On the other side, Multi-point methods allow the user not to throw away the previously computed information and use them to calculate the next point. Multipoint techniques belong to the class of the most efficient root-finding techniques for nonlinear problems [3]. The ability of the Multi-point methods to overcome theoretical limits of one-point methods, prompted an interest in developing these methods [4]. Despite the proper development of Multi-point methods, there are very few studies for application of them in engineering nonlinear problems. A Multi-point method that needs only one evaluation of the Jacobian matrix per iteration was presented by Arroyo et al. [5], this fifth order method was used for approximation of artificial satellites' preliminary orbits. Kiran et al. [6] utilized six numerical methods instead of Newton's method to solve the nonlinear equations in the Gurson plasticity and J2 plasticity constitutive models. Derakhshandeh and Pourbagher [7] used Newton-like methods to solve power flow equations. Kiran and Khandelwal [8] also tested the performance of numerical methods for solving nonlinear equations in fracture analysis.

New developments in digital computers have brought enthusiasm in the non-linear structural problems for over three decades. A variety of numerical methods have been presented to compute geometrically nonlinear equilibrium paths. As it was first proposed by Riks [9] and Wempner [10], for tracing the equilibrium paths past the first limit point, the constraint equation is needed to complement the equilibrium equations. The normal plane to the tangent plane was proposed independently by them as a mixed load-displacement constraint equation. Ramm [11] used this method by updating the normal plane at each iteration. Crisfield [12] presented a faster method with respect to the previous

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procedures utilizing an n-dimensional sphere as a constraint. Using the quadratic constraint equation and line searches techniques, an automatic incremental method was later extended by the same author [13]. Schweizerhof and Wriggers [14] proposed a method which always provides a real and unique solution to be used whenever the roots of the Crisfield's constraint equation is complex. As remarked by Carrera [15], although the linearized method of Schweizerhof and Wriggers [14] could be proper with respect to the methods offered by Riks [9], Wempner [10] and Ramm [11], it is inferior to Crisfield's quadratic method. Bellini and Chullya [16] introduced an algorithm utilizing some of the essential ideas from that of Bathe and Dorkin [17] to improve Crisfield's method. The general stiffness parameter is used by Yang and Shieh [18] to automatically control the load increment. Using orthogonality principles, Forde and Stiemer [19] suggested a general arc-length method. In order to non-dimensionalizing the vectors that define the constraint equations, a scaled arc-length procedure is presented by Al-Rasby [20]. The geometrical interpretation of the arc-length techniques is given by Fafard and Massicotte [21]. They also proposed the modified Crisfield-Ramm method [21]. Carrera [15] presented two novel algorithms based on Riks [9] and Wempner [10] method. To improve the Crisfield's method, Carrera [15] also proposed to choose the roots of the quadratic constraint equations closer to the solution of the linearized method of Schweizerhof and Wriggers [14]. The Carrera's algorithms implemented and discussed later in Refs. [22–25]. Zhou and Murraya [26] proposed a modified formulation of the arc-length control criterion using partial corrections to avoid complex roots. Using the sign of determinant of the current stiffness matrix, Feng et al. [27] presented a procedure to compute the initial load increment. Szyszkowski and Husband [28] proposed the curvature controlled arc-length method using a new geometrical explanation of the arc-length technique. A self-adaptive arc-length method introduced using a prescribed cone of admissible directions in equilibrium path is presented by Ligarò and Valvo [29]. The arc-length techniques in a unified way with geometric interpretations are provided by Ritto-Correa and Camotim [30]. The capabilities of the various techniques are examined through numerical examples by Leon et al. [2]. Recently, new researches have been conducted to find new techniques to follow the equilibrium path. A comparative analysis on the performance of two iterative numerical methods for geometrically nonlinear analysis of trusses before the first limit point, was studied by Mahdavi et al. [31]. The applicability of the multistage homotopy perturbation method in nonlinear dynamic analysis of space trusses was investigated by Shon et al. [32]. A class of arc-length methods for delamination problems which combines geometric and dissipative constraint equations is presented by Bellora and Vescovini [33]. To improve the robustness and the efficiency of Newton method in geometrically nonlinear analysis, Magisano et al. [34] proposed a novel strategy, called MIP Newton. A modified Newton-type Koiter-Newton method to follow the geometrically nonlinear paths of structures is proposed by Liang et al. [35]. A path following method for finding the correct equilibrium state for general systems is presented by Rose et al. [36]. Groh and Pirrera [37] presented a generalized path following technique which combines a continuation method with the geometrical versatility of the finite element method for the analysis and design of well-behaved structures. An algorithm based on bifurcation-detection method and Koiter-Newton method for equilibrium path tracing of thin-walled shells is presented by Liang and Sun [38].

Note that this article focuses on geometrical nonlinearity. The main purposes of this paper are, (1) presentation of algorithms based on the Multi-point methods for the nonlinear analysis, (2) investigation of the robustness and efficiency of four Multi-point iterative methods in the geometrically nonlinear analysis of structures, (3) to propose four novel algorithms by applying a modification to compute nonlinear equilibrium paths beyond limit points, (4) a comparison of numerical results obtained from Multi-point methods and the proposed techniques with exact solutions, (5) application of Multi-point methods and the modified ones in the geometrically nonlinear analysis of domes and shells.

2. Application of Multi-point methods for solving nonlinear equilibrium equations

Higher derivatives of the function are needed to achieve high order in one point methods and from a computational point of view, calculation of higher derivatives is expensive. Multi-point methods are efficient root-finding techniques that do not throw away information previously computed. This approach provides the development of the most efficient root finding methods, which caused recent interest in the construction of Multi-point iterative methods [4].

Let $F(x) = 0$ is a system of n-nonlinear equations. In these equations x is a vector with n global variables and, $F'(x)$ is the first derivative (also called Jacobian matrix) of $F(x)$. A two-step Mult-point method was developed by Homeier [39] for multivariate case, which can be given as

$$x^{(k+1)} = x^{(k)} - F'\left(x^{(k)} - \frac{1}{2}F'(x^{(k)})^{-1}F(x^{(k)})\right)^{-1}F(x^{(k)}). \tag{1}$$

This method was earlier presented as a modification of the Newton method for finding the root of a univariate function [40].

A cubic Mult-point method was proposed by Weerakoon and Fernando [41] for multivariate case, is given as

$$x^{(k+1)} = x^{(k)} - 2\left[F'\left(x^{(k)} - \frac{1}{2}F'(x^{(k)})^{-1}F(x^{(k)})\right) + F'(x^{(k)})\right]^{-1}F(x^{(k)}). \tag{2}$$

Weerakoon and Fernando derived this Multi-point method applying numerical integration [41], whereas this method was originally proposed by Traub [42].

The fourth order Generalized Jarratt's method [43] is given as

$$^{*}x^{(k)} = x^{(k)} - \frac{2}{3}F'(x^{(k)})^{-1}F(x^{(k)}), \tag{3}$$

$$x^{(k+1)} = x^{(k)} - \frac{1}{2}\left[3F'(^{*}x^{(k)}) - F'(x^{(k)})\right]^{-1}\left[3F'(^{*}x^{(k)}) + F'(x^{(k)})\right]F'(x^{(k)})^{-1}F(x^{(k)}). \tag{4}$$

At first, this fourth order procedure was proposed by Jarratt [43] for univariate case. Later, this method used for multivariate cases in many studies [44,45].

The fourth order Darvishi and Barati method [46] is defined as

$$^{*}x^{(k)} = x^{(k)} - F'(x^{(k)})^{-1}\left(F(x^{(k)}) + F(x^{(k)} - F'(x^{(k)})^{-1}F(x^{(k)}))\right), \tag{5}$$

$$x^{(k+1)} = x^{(k)} - \left[\frac{1}{6}F'(x^{(k)}) + \frac{2}{3}F'\left(\frac{x^{(k)} + {}^{*}x^{(k)}}{2}\right) + \frac{1}{6}F'(^{*}x^{(k)})\right]^{-1}F(x^{(k)}). \tag{6}$$

This fourth order method is a modification of the third-order methods based on quadrature formulae proposed by Fortini and Sormani [47].

Nonlinear analysis of structures consists of solving the system of nonlinear equations using incremental/iterative strategies. In the case that the structure is under a specific external load vector which is $\mathbf{R}^{ext}$ the system nonlinear equilibrium equations can be written as

$$\mathbf{R}(\mathbf{a}) = \mathbf{R}^{\text{int}}(\mathbf{a}) - \mathbf{R}^{ext} = 0 \tag{7}$$

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