Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Extension of dynamic stiffness method to complicated damped structures

Han Fei, Dan Danhui *, Cheng Wei

School of Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China

ARTICLE INFO

Article history:
Received 22 May 2018
Accepted 30 July 2018
Available online xxxx

Keywords:
Exact dynamic analysis
Improved dynamic stiffness method
Wittrick-Williams algorithm
Damped structures
Viscous and hysteresis damping

ABSTRACT

The dynamic stiffness method is an exact method for structural dynamic analysis. By separating the variable of the displacement function in frequency domain, the dynamic stiffness matrix and frequency equation of the structure are obtained, and the structural dynamic analysis can then be achieved by solving the transcendental frequency equation. For undamped systems, the frequency equation can be accurately solved by the Wittrick-Williams algorithm, however, the frequency equation of damped structures is a complex transcendental equation and many root-search techniques performing well in real field including the Wittrick-Williams algorithm are no longer applicable. Therefore, the application of dynamic stiffness in damped structures is a major challenge and has not been well resolved. In view of this, aiming at the classically damped system in the project, this paper has improved the dynamic stiffness method from two aspects, (1) The calculation principle. By performing the variable separation in Laplace domain instead of frequency domain, this paper established the relationship between damped frequency and undamped frequency by a proposed method for the calculation of the damping ratio, thus avoiding the solution of the hard-to-solve complex transcendental frequency equation; (2) The solution method. To make the method widely applicable, an improved Wittrick-Williams algorithm is given in this paper to solve the frequency equation of complicated systems. Finally, numerical examples are used to verify the accuracy and universality of the proposed method.

1. Introduction

The dynamic stiffness method (DSM) relating the amplitudes of applied forces and responses of a harmonically vibrating continuum has been widely studied and applied in recent decades [1]. The success of the method is due to the fact that the structure only needs to be specifically divided at geometric or material discontinuities, thus a few elements can be applied to calculate any order modes without discretizing the structure. This property of DSM makes it suitable for arbitrary boundary conditions and frequency ranges, especially effective in solving high-precision and high-order modes for continuous structures of beams and plates [2]. Arches [3], frames [4], and prismatic plate structures [5] have already be studied in the past several decades. In recent years, the method has been applied to composite beam [6–8] and plate structures [9,10].

However, the method has been applied almost exclusively to undamped structures whose oscillations are harmonic, or periodic [11]. This is due mainly to two factors, one is the rather misleading intuition that only harmonic vibrations can be described by solutions with separate time- and space-dependent factors; the other is the inherent problems of dynamic stiffness method, namely the difficulties in solving the transcendental frequency equation. To overcome these difficulties, the Wittrick–Williams (W-W) algorithm was put forward in 1970 and has been widely used in many one-dimensional structures, such as bars, frames, and beams, which can not only obtain the roots of the frequency equation with the required precision, but also perfectly solve the root-missing problem [12]. However, for damped or complicated systems, the original DSM and W-W theories will no longer work [13]. In view of this, the main work of this paper centres on how to make the DSM and W-W algorithm still applicable to complicated damped systems, and an extended dynamic stiffness method (EDSM) which is capable for a much wider class of problem of classically damped structures is proposed in this paper. The main improvements of this method can be illustrated in two aspects.

(1) The first improvement is starting from the basic principles of the DSM, and the complex frequency parameters are used instead of the frequency parameters in original DSM when

* Corresponding author at: Room 711, Bridge Building, Tongji University, 1239 Siping Road, Shanghai, China.

E-mail addresses: dandanhui@tongji.edu.cn (D. Dan), chengwei@tongji.edu.cn (W. Cheng).

https://doi.org/10.1016/j.compstruc.2018.07.010
0045-7949/

Please cite this article in press as: Han F et al. Extension of dynamic stiffness method to complicated damped structures. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.07.010

preforming the variable separation. The convenience of such processing is able to avoid the appearance of complex elements in the dynamic stiffness matrix, and it can also establish the relation between the damped frequency and undamped frequency by calculating the damping ratio of the system with some techniques, thus avoiding the solution of the hard-to-solve complex transcendental frequency equation;

(2) The second improvement is to make the W-W algorithm applicable to complicated structures. Under the premise of avoiding solving the complex transcendental equations, the W-W algorithm can continue to exert its powerful computational advantages. However, the original W-W algorithm can only be used to some simple structures or at specific boundary conditions. Based on this, this paper proposed an improved Wittrick-Williams algorithm, which can be used to calculate the modal frequency of complex structures with arbitrary boundary conditions.

To illustrate the universality of the method, both viscous damping and hysteretic damping system are analyzed in the principle section. The accuracy of the proposed EDSM are verified by numerical examples.

2. Fundamental principle

2.1. Calculation process of original dynamic stiffness method

DSM is usually applied to the dynamic analysis of undamped systems, during the solution process, it is firstly need to introduce the harmonic vibration assumption, that is, to transform the displacement functions of the system from the time domain to frequency domain; Then the general solution of the governing differential equation (GDE) in frequency domain and the time-independent displacement amplitude are obtained by variables separation; Finally, combined with the nodal force and displacement boundary conditions, the dynamic matrix of the system is obtained by eliminating the undetermined coefficients in the general solution. In general, the free vibration of an undamped structure can be described by the following differential equations [14]:

$$L(\mathbf{u}) = 0 \tag{1}$$

where L is the differential operator and $\mathbf{u}$ is the corresponding displacement vector. By introducing the simple harmonic motion assumptions, the displacement $\mathbf{u}$ can be expressed as:

$$\mathbf{u} = \mathbf{U}e^{i\omega t} \tag{2}$$

where $\mathbf{U}$ is the displacement amplitude vector, ω is the circular frequency (rad/s), t is the time, $i = \sqrt{-1}$.

By substituting Eq. (2) into Eq. (1), the time-related terms will be eliminated and yields

$$L_1(\mathbf{U}, \omega) = 0 \tag{3}$$

where L_1 is the differential operator. The solution of Eq. (3) can be determined by

$$\mathbf{U} = \mathbf{AC} \tag{4}$$

where $\mathbf{C}$ is the coefficient vector, $\mathbf{A}$ is a square matrix related to the frequency.

Then, by substituting Eq. (4) into the displacement boundary condition, the nodal displacement vector $\boldsymbol{\delta}$ can be expressed as

$$\boldsymbol{\delta} = \mathbf{BC} \tag{5}$$

where matrix $\mathbf{B}$ is a square matrix obtained from $\mathbf{A}$ after substituting Eq. (4) into the displacement boundary conditions.

Then, the relationship between the nodal force and the vector $\mathbf{C}$ can also be obtained by introducing the force boundary condition

$$\mathbf{F} = \mathbf{DC} \tag{6}$$

where $\mathbf{F}$ is the nodal force vector, $\mathbf{D}$ is the square matrix related to the frequency. After eliminating the constant vector $\mathbf{C}$ from Eqs. (5) and (6), we have

$$\mathbf{F} = \mathbf{DB}^{-1}\boldsymbol{\delta} = \mathbf{K}\boldsymbol{\delta} \tag{7}$$

where $\mathbf{K} = \mathbf{DB}^{-1}$ is the dynamic stiffness matrix. For free vibration, the dynamic equilibrium equation Eq. (7) becomes

$$\mathbf{K}\boldsymbol{\delta} = \mathbf{0} \tag{8}$$

Eq. (8) is a homogeneous equation, usually, to obtain a nontrivial solution yield

$$|\mathbf{K}(\omega)| = \mathbf{0} \tag{9}$$

where $|\cdot|$ represents the value of the determinant. The frequency satisfying the characteristic equation Eq. (9) (that is, the frequency value at the zero point of $|\mathbf{K}(\omega)|$) is the natural frequency of the structure.

2.2. Principles of extended dynamic stiffness method

When the DSM is employed to analyze a damped system, the existence of the damping term in the GDE will lead to a complex dynamic stiffness matrix, and then the frequency equation will become a complex transcendental equation, making it difficult to solve. The calculation process of the EDSM is basically the same as that of DSM, except that the displacement vector and load vector of the structure need to be converted from the time domain to the Laplace domain rather than frequency domain when performs the variables separation. Then the GDE and the frequency equation $|\mathbf{K}(\lambda)| = \mathbf{0}$ of the system in the Laplace domain can be obtained, where $\lambda = \alpha + i\beta$, $i = \sqrt{-1}$. This method can not only effectively avoid the generation of complex elements in dynamic stiffness matrix, but also does not increase the difficulty in solving the frequency equation. In addition, the hysteretic damping and viscous damping can be easily considered by EDSM when analyzing the damped system. In particular, for a damped system there is $\alpha = 0$ and $\lambda = i\omega$.

To introduce the calculation steps of EDSM in different damped systems, the Euler beam is taken as an example to illustrate the application in viscous and hysteretic damped system.

2.2.1. Undamped system

Considering the undamped free vibration of a uniform Euler beam, whose GDE is [1]

$$EI\frac{\partial^4 u}{\partial x^4} + m\frac{\partial^2 u}{\partial t^2} = 0 \tag{10}$$

where EI and m are the flexure stiffness and mass per unit length of the beam separately, $u(x,t)$ is the transverse displacement function. Assuming that the solution has the form of $u(x,t) = U(x)e^{\lambda_0 t}$, then Eq. (10) yields

$$EI\frac{\partial^4 U}{\partial x^4} + \lambda^2 mU = 0 \tag{11}$$

Let $a_0 = 1$ and $b_0 = \lambda_0^2$, the equation above can be rewritten as a general form as

$$a_0 EI\frac{\partial^4 U}{\partial x^4} + b_0 mU = 0 \tag{12}$$

Suppose the general form of the above equation is $U = Ae^{\kappa x}$, then the characteristic function of Eq. (12) can be expressed as