



A world population growth model: Interaction with Earth's carrying capacity and technology in limited space

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ABSTRACT

Up to 1900, world population growth over 1500 years fitted the quasi-hyperbolic format $P(t) = a/(D - t)^M$, but this fit projected to infinite population around 2000. The recent slowdown has been fitted only by iteration of differential equations. This study fits the mean world population estimates from CE 400 to present with “tamed quasi-hyperbolic function” $P(t) = A/[\ln(B + e^{(D-t)/\tau})]^M$, which reverts to $P = a/(D - t)^M$ when $t \ll D$. With coefficient values $P(t) = 3.83 \times 10^9 / [\ln(1.28 + e^{(1980-t)/22.9})]^{0.70}$, the fit is within $\pm 9\%$, except in 1200–1400, and projects to a plateau at 10.2 billion. An interaction model of population, Earth's carrying capacity and technological–organizational skills is proposed. It can be approximately fitted with this $P(t)$ and an analogous equation for carrying capacity.

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1. Introduction

Fifty years ago, *Science* published a study with the provocative title “Doomsday: Friday 13 November, A.D. 2026” [1]. It fitted world population during the previous two millennia with $P = 179 \times 10^9 / (2026.9 - t)^{0.99}$. This “quasi-hyperbolic” equation (hyperbolic having exponent 1.00 in the denominator) projected to infinite population in 2026 – and to an imaginary one thereafter. Later growth has fallen short of this equation, calling for a modification that averts “doomsday”. The smoothness of world population growth curve since CE 400, with a single inflection point around 2000, suggests that stable long-term factors may be at work, rather than accumulation of random developments. This underlying basis for quasi-hyperbolic pattern and later slowdown needs elaboration. Fits based on iterations of differential equations have been offered [2,3], but no explicit function $P(t)$ like the one above.

Here a modified explicit equation is proposed, which fits the mean world population estimates from CE 400 to present and to foreseeable future. This “tamed quasi-hyperbolic function” fits approximately an interaction model of population, Earth's

carrying capacity and technological–organizational skills. This three-factor model combines two earlier ones, one in terms of population and technology only [4–6], and the other in terms of population and Earth's human carrying capacity only [2,7]. Application to other phenomena with apparent asymptotes is briefly discussed, as well as population growth outside the time period considered.

2. Quasi-hyperbolic growth up to 1900

Over the last 1600 years human population has increased 35-fold. Up to the mid-1900s, it grew at an ever-increasing percent rate per year, which the exponential model cannot express. As early as 1951, André de Cailleux [8] noticed that the world population fitted a quasi-hyperbolic equation:

$$P = a/(D-t)^M \quad (1)$$

where a , D and M are constants. In differential form,

$$dP/dt = Ma/(D-t)^{M+1} = (M/a^{1/M})P^{1+1/M} \quad (2)$$

and the relative growth rate is simply $dP/Pdt = M/(D - t)$. At asymptote $t = D$, $P \rightarrow \infty$. Eq. (1) becomes purely hyperbolic

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when $M = 1$ and exponential when $M \rightarrow \infty$. In 1958–1979, this pattern was repeatedly rediscovered or confirmed [1,4,9–12], right at the time when world population began to fall visibly short of this pattern. The estimates for D ranged from 2005 to 2027 [1,4,9–12]. In retrospect these were overestimates, because they included post-1900 data, when the shift away from the quasi-hyperbolic pattern had already set in, however mildly. Correspondingly, exponent M has been also overestimated, ranging from 0.74 up to 1. By habit, CE 1 has most often been taken as the starting point for data fitting, but this was the high point of a previous speedup and leveling-off in world population growth, to be discussed later. A new smooth upward trend in population began around CE 400. The best fit for population estimates from CE 400 to 1900 is close to

$$P_Q = 34.3 \times 10^9 / (1980 - t)^{0.70} \tag{3}$$

where subscript Q indicates quasi-hyperbolic fit. Mean estimates of world population during these 1500 years (Table 1) agree with Eq. (3) within $\pm 5\%$, except for spurts in 1200–1300 and 1850, and shortfall in 1400 (Black Death).

Table 1

World population from CE 400 to 2010, in million: mean and range of estimates, and as calculated from quasi-hyperbolic and T-function approaches. Shown in bold are cases where values from Eq. (15) fall outside the range of estimates by more than $\pm 3\%$. Time intervals are taken so that successive population ratios remain between 1.1 and 1.3.

Year	Estimates of P		P_Q from Eq. (3)	P_T from Eq. (15)	Deviation (%) of mean from Eq. (15)
	Mean	Range			
400	198	190–206	197.8	197.8	+0.1
600	214	200–237	217.5	217.5	–1.6
800	235	220–261	242.5	242.5	–3.2
1000	281	254–310	276.3	276.3	+1.7
1100	310	301–320	297.8	297.8	+4.1
1200	398	360–450	324.2	324.2	+22.8
1300	396	360–432	356.9	356.9	+11.0
1400	362	350–374	398.9	398.9	–10.2
1500	457	425–500	455.4	455.4	+0.3
1600	544	498–579	536.3	536.1	+1.5
1700	635	603–679	664.2	663.9	–4.6
1750	771	720–824	762.2	761.8	+1.2
1800	941	890–981	904.9	904.5	+4.0
1850	1242	1200–1265	1136.4	1135.5	+9.3
1900	1639	1564–1680	1596.4	1583.5	+3.5
1920	1905	1860–1968	1952	1906	–0.1
1940	2313	2300–2340	2593	2401	–3.8
1950	2526	2499–2556	3172	2748	–8.8
1960	3035	3023–3042	4213	3185	–4.9
1970	3667	3600–3712	6844	3728	–1.7
1980	4442	4436–4453	∞	4385	+1.3
1990	5278	5260–5290		5147	+2.5
2000	6021	5750–6115		5980	+0.7
2010	6861	6831–6909		6825	+0.5

Mean estimates and ranges of world population are those of mean estimates by 9 sources in Wikipedia, http://en.wikipedia.org/wiki/World_population_estimates, visited 11/9/10: US Census Bureau 2009; Population Reference Bureau 2008; UN Dept. of Econ. and Soc. Aff. 2008; HYDE 2006, A. Maddison 2003; J. H. Tanton 1994; J.-N. Biraben 1980; C. McEvedy and R. Jones 1978, R. Thomlinson 1975; J. D. Durand 1974; Clark 1967. Estimates given with only 100 million precision were omitted. During the period since CE 400, individual estimates deviate from the means by up to 13%. Evaluating the validity of population estimates in centuries past is beyond the scope of this study.

This millennial upward curvatures in $\log P(t)$ and the recent downward curvature are quite disparate. To fit them together, one has to consider deep-set factors boosting growth and now slowing it down. Differential equations can be set up, and an iteration process can be used to fit actual population estimates [2,3], but integration into a single explicit equation has been lacking. An ultimate ceiling (U) could easily be inserted into Eq. (2):

$$dP/dt = KP^{1+1/M}(1-P/U). \tag{4}$$

Due to $P^{1/M}$, this is not simple exponential approach to the ceiling, but it satisfies the basic desiderata – quasi-hyperbolic initial growth plus a ceiling. No integration formula is available, however, for non-integer M . The same applies to $dP/dt = a / [(D - t)^2 + c^2]$, also proposed [13]. Attempts have been made to use functions $P(t)$ other than quasi-hyperbolic, as reviewed in [14], but it is hard to match its simplicity and degree of fit prior to 1900.

What could cause the quasi-hyperbolic pattern, and the later slowdown? At least two other factors must interact with population. Some models have focused on technology (T) [4–6], others on Earth's carrying capacity (C) at a given time [2,3]. Space limitations on Earth may impose an ultimate population limit even when technology may uncover new resources. Dry land area is the ultimate resource, upon which most others are predicated. A ceiling at $U = 10$ billion would mean 1.5 ha per person. If evenly spread out, with 940 million placed in Antarctica, humans would stand at about 100 m from their six closest neighbors. Two models are reviewed next, respectively based on technology [4] and on carrying capacity [2]. Weak links are pointed out in each, and a new model subsuming both will be presented.

3. Interaction with technology and carrying capacity

3.1. The population–technology model

Assume endogenous exponential growth of population. Assume the same for “technology” [15], using this term in its broadest meaning, which includes social organization skills as stressed in the world systems literature [16]. Indeed, “skills” might express this broad ability more clearly [12], but we'll stick with the traditional term. Eq. (1) can be derived from interaction between these exponential growths when they reciprocally enhance their rate “constants” [4]. Such interaction might be assumed because more people means more potential innovators, and higher technological–organizational skills increase Earth's carrying capacity and hence make a larger population possible. Assume that interaction terms can be approximated by power functions [4].

Then

$$dP/dt = kT^n P \tag{5}$$

$$dT/dt = hP^m T \tag{6}$$

Eliminating time, P and T are related as $dP/dT = (k/h) P^{1-m}/T^{1-n}$. Integration yields $hnP^m = kmT^n$, when we assume that $T = 0$ when $P = 0$. Inserting $kT^n = hnP^m/m$ into Eq. (5) leads to $dP/dt = (hn/m)P^{m+1}$, which is equivalent to

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