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Cyclotomic p-adic multi-zeta values

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ABSTRACT

The cyclotomic p-adic multi-zeta values are the p-adic periods of $\pi_1^{uni}(\mathbb{G}_m \setminus \mu_M, \cdot)$, the unipotent fundamental group of the multiplicative group minus the M-th roots of unity. In this paper, we compute the cyclotomic p-adic multi-zeta values at all depths. This paper generalizes the results in [9] and [10]. Since the main result gives quite explicit formulas we expect it to be useful in proving non-vanishing and transcendence results for these p-adic periods and also, through the use of p-adic Hodge theory, in proving non-triviality results for the corresponding p-adic Galois representations.

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1. Introduction

There are not many examples of motives over $\mathbb{Z}$. The most basic examples of such motives are the Tate motives. Another one is the unipotent completion $\pi_1^{uni}(\mathbb{G}_m \setminus \{1\}, \cdot)$ of the fundamental group of the thrice punctured projective line at a suitable tangential basepoint [2]. In fact by a theorem of F. Brown, this motive generates the tannakian category of mixed Tate motives over $\mathbb{Z}$. The complex periods of $\pi_1^{uni}(\mathbb{G}_m \setminus \{1\}, \cdot)$ are $\mathbb{Q}$-linear combinations of the multi-zeta values given by

$$\zeta(s_1, s_2, \cdots, s_k) := \sum_{0<n_1<\cdots<n_k} \frac{1}{n_1^{s_1} n_2^{s_2} \cdots n_k^{s_k}},$$

for $s_1, \cdots, s_{k-1} \geq 1$ and $s_k > 1$. These values were defined by Euler for depth less than or equal to 2 and by Écalle in full generality [4, p. 429] and studied further by Deligne, Goncharov, Terasoma, Zagier etc.

Similarly, one can consider the unipotent fundamental group $\pi_1^{uni}(\mathbb{G}_m \setminus \mu_M, \cdot)$ of the multiplicative group minus the group μ_M of M-th roots of unity for $M \geq 1$. If $\mathcal{O}_M$ denotes the ring of integers of the M-th cyclotomic field, then this fundamental group defines a mixed Tate motive over $\mathcal{O}_M[1/M]$. The periods of this motive are linear combinations of the cyclotomic multi-zeta values

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$$\sum_{0<n_1<\cdots<n_k} \frac{\zeta^{i_1 n_1+\cdots i_k n_k}}{n_1^{s_1} n_2^{s_2} \cdots n_k^{s_k}},$$

where i_j, for $1 \leq j \leq k$, are fixed integers and ζ is an M-th root of unity. These values were studied and related to modular varieties and the theory of higher cyclotomy in [6].

This paper concerns the p-adic periods of the motive $\pi_1^{uni}(\mathbb{G}_m \setminus \mu_M, \cdot)$. For any number field E, and any non-archimedean place ν of E, we define a realisation map from the category of mixed Tate motives over E to the category of mixed Tate filtered (φ, N)-modules over E_ν [1], the completion of E at ν. Also to any (framed) mixed Tate filtered (φ, N)-module over E_ν, we associated a p-adic period [1]. If we apply this construction to the motive $\pi_1^{uni}(\mathbb{G}_m \setminus \mu_M, \cdot)$ and the number field $\mathbb{Q}[\zeta]$, where ζ is a primitive M-th root of unity, we see that the cyclotomic p-adic multi-zeta values are the p-adic periods associated to the mixed Tate motive defined by the unipotent fundamental group of $\mathbb{G}_m \setminus \mu_M$, for $p \nmid M$. These values were defined in terms of the action of the crystalline frobenius on the fundamental group in [9], generalising the notion of p-adic multi-zeta values in [8]. In this paper we give an explicit series representation of these p-adic periods. This is a generalisation of [10] to the cyclotomic case.

We give an overview of the contents of the paper. In §2, we start with studying certain types of series in terms of which the cyclotomic p-adic multi-zeta values will be expressed. These series can be of two types, denoted by σ or γ, and are called the *cyclotomic p-adic iterated sum series* (or *ciss*). In fact the *ciss* are divergent and we will need to regularise them. The regularisation can be intuitively thought of as removing a combination of the summands which have large p factors in the denominators that cause divergence. More precisely, we extend the algebra of M-power series functions by adding some highly divergent functions which we denote by σ_p and we show in Proposition 2.9 that the *ciss* are contained in this algebra. In Corollary 2.6, we show that the $\{\sigma_p(\underline{s};\underline{i})\}$'s form a basis for this extended algebra as a module over the algebra of M-power series functions. These two facts help us to define the regularised versions of the *ciss*, denoted by $\tilde{\sigma}$ and $\tilde{\gamma}$, in Definition 2.10. The limits of these regularised series are called the *cyclotomic p-adic iterated sums* (or *cis*), and denoted by $\underline{\sigma}$ and $\underline{\gamma}$. Let ζ be a primitive M-th root of unity. Let $\mathcal{P}_M$ denote the $\mathbb{Q}(\zeta)$-algebra generated by the *cis*, and $\mathcal{Z}_M$ the algebra generated by the cyclotomic p-adic multi-zeta values. In §3, we collect the results and recall the concepts and notations that we will use from [9]. The main theorem is

Theorem 1.1. *We have the inclusion* $\mathcal{Z}_M \subseteq \mathcal{P}_M$.

The proof of this theorem occupies the whole of §4. The proof expresses in an inductive way every cyclotomic p-adic multi-zeta value as a series and should be thought of as an explicit computation of these values.

We would like to mention that Furusho defined in [5] another p-adic version of multi-zeta values that is essentially equivalent to ours in [8]. More precisely, the two versions generate the same algebra and each version can be obtained from the other one by elementary linear algebraic manipulations. This is explained in detail in [10, Lemma 3.13]. One can also define a version of cyclotomic version of Furusho's p-adic multi-zeta values which will again be essentially equivalent to the above version by the proof of [10, Lemma 3.13]. Finally, we would also like to mention that D. Jarossay has a different explicit expression for cyclotomic p-adic multi-zeta values in [7] obtained independently except for the dependence on [8] and [9].

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