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Rigid cohomology via the tilting equivalence

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ABSTRACT

We define a de Rham cohomology theory for analytic varieties over a valued field $K^\flat$ of equal characteristic p with coefficients in a chosen untilt of the perfection of $K^\flat$ by means of the motivic version of Scholze's tilting equivalence. We show that this definition generalizes the usual rigid cohomology in case the variety has good reduction. We also prove a conjecture of Ayoub yielding an equivalence between rigid analytic motives with good reduction and unipotent algebraic motives over the residue field, also in mixed characteristic.

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1. Introduction

Rigid cohomology can be considered as a substitute of the pathological (at least for non-proper varieties) de Rham cohomology over a field k of positive characteristic. Its definition is based on the idea of associating to a variety $\bar{X}/k$ another variety X over a field K of characteristic 0 and then considering its well-behaved de Rham cohomology.

The classic path to reach this goal, elaborating on the work of Monsky and Washnitzer, is to find a smooth formal model $\mathfrak{X}$ of $\bar{X}$ over a valuation ring $\mathcal{O}_K$ of mixed characteristic, then to consider its (rigid analytic) generic fiber X.

From Fontaine's and Scholze's work in p-adic Hodge theory, we are now accustomed with another strategy to change characteristic: the so-called *tilting equivalence*. This equivalence is built between perfectoid spaces (a certain kind of adic spaces, see [21]) over a perfect, non-archimedean field $K^\flat$ of characteristic p, and perfectoid spaces over a fixed *untilt* K of $K^\flat$. We recall that such *untilts* are perfectoid fields (that is, complete with respect to a non-discrete valuation of rank 1, with residue characteristic equal to $p > 0$ and such that the Frobenius map is surjective over $\mathcal{O}_K/p$) of mixed characteristic, have the same absolute Galois group as $K^\flat$ and are parametrized by principal ideals of $W(\mathcal{O}_{K^\flat})$ generated by a primitive element of degree one (see [14] and [21]). Needless to say, this perfectoid framework of Scholze has been reaping many interesting results in p-adic Hodge theory, and in arithmetic geometry in general (see [22]).

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An alternative method for making $\bar{X}$ "change characteristic" is then at reach: we can base change $\bar{X}$ to a perfectoid field $K^\flat$ then we can take its analytification, followed by its perfection. This defines a perfectoid space $\widehat{X}^\flat$ over $K^\flat$ that we can finally switch (by Scholze's equivalence) to a perfectoid space $\widehat{X}$ over a chosen untilt K of $K^\flat$ having characteristic 0.

The initial problem, namely the definition of a de Rham cohomology for $\bar{X}$, is not yet solved with the above procedures. Indeed, for rigid analytic varieties as well as for perfectoid spaces, the de Rham complex is still problematic (its cohomology groups can be oddly infinite-dimensional for smooth, affinoid rigid analytic varieties). Nonetheless, the results of [15] and [23] show that some natural de Rham cohomology groups for both smooth rigid analytic varieties as well as smooth perfectoid spaces can truly be defined.

One aim of this paper is that the two recipes mentioned above are actually equivalent. We now rephrase this statement in terms of motives. This allows for a more precise result, and further corollaries that we examine afterwards.

Out of the categories of smooth varieties over a field, smooth rigid analytic varieties over a non-archimedean field, smooth perfectoid spaces over a perfectoid field, or smooth formal schemes over a valuation ring, one can construct the associated category of motives, written as $\mathbf{DM}(K)$ (or $\mathbf{RigDM}(K)$, $\mathbf{PerfDM}(K)$ and $\mathbf{FormDM}(\mathcal{O}_K)$ in the various cases). We consider here derived, effective motives with rational coefficients following Voevodsky. In particular, these categories are Verdier quotients of the classical derived categories of étale sheaves with $\mathbb{Q}$-coefficients defined on each big site.

The special fiber and the generic fiber functors on varieties induce some functors also at the level of motives:

$$\xi\colon \mathbf{DM}(k) \overset{\sim}{\leftarrow} \mathbf{FormDM}(\mathcal{O}_K) \to \mathbf{RigDM}(K)$$

(the first one has a natural quasi-inverse as proven in [6]). The composition from left to right is then the motivic version of the first recipe (Monsky–Washnitzer's) sketched above.

Suppose now K is perfectoid. It is possible to rephrase Scholze's tilting equivalence in motivic terms by saying that the categories $\mathbf{PerfDM}(K)$ and $\mathbf{PerfDM}(K^\flat)$ are equivalent, where $K^\flat$ is its unique *tilt* in characteristic p (see [21]). In [23] we descended this result to the rigid analytic situation, by proving the following:

Theorem *([23]). Let K be a perfectoid field. There is a canonical monoidal, triangulated equivalence*

$$\mathfrak{G}\colon \mathbf{RigDM}(K) \cong \mathbf{PerfDM}(K).$$

In particular, for any perfectoid field K of mixed characteristic, we obtain a canonical monoidal, triangulated equivalence

$$\mathfrak{G}_{K,K^\flat}\colon \mathbf{RigDM}(K) \cong \mathbf{RigDM}(K^\flat).$$

We can then consider the following functors

$$\xi^\flat\colon \mathbf{DM}(k) \to \mathbf{DM}(K^\flat) \xrightarrow{\mathrm{Rig}^*} \mathbf{RigDM}(K^\flat) \cong \mathbf{RigDM}(K)$$

and their composition corresponds to the second recipe that we sketched above. We then prove (see Theorem 3.2 and Proposition 3.5):

Theorem. *Let K be a perfectoid field. The following diagram commutes, up to an invertible natural transformation.*

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