Accepted Manuscript

Homomorphisms between different quantum toroidal and affine Yangian algebras

Mikhail Bershtein, Alexander Tsymbaliuk

PII: S0022-4049(18)30126-9
DOI: https://doi.org/10.1016/j.jpaa.2018.05.003
Reference: JPAA 5907

To appear in: *Journal of Pure and Applied Algebra*

Received date: 24 January 2018
Revised date: 13 April 2018

Please cite this article in press as: M. Bershtein, A. Tsymbaliuk, Homomorphisms between different quantum toroidal and affine Yangian algebras, *J. Pure Appl. Algebra* (2018), https://doi.org/10.1016/j.jpaa.2018.05.003

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

HOMOMORPHISMS BETWEEN DIFFERENT QUANTUM TOROIDAL AND AFFINE YANGIAN ALGEBRAS

MIKHAIL BERSHTEIN AND ALEXANDER TSYMBALIUK

Abstract. This paper concerns the relation between the quantum toroidal algebras and the affine Yangians of $\mathfrak{sl}_n$, denoted by $\mathcal{U}^{(n)}_{q_1,q_2,q_3}$ and $\mathcal{Y}^{(n)}_{h_1,h_2,h_3}$, respectively. Our motivation arises from the milestone work [GTL], where a similar relation between the quantum loop algebra $U_q(L\mathfrak{g})$ and the Yangian $Y_h(\mathfrak{g})$ has been established by constructing an isomorphism of $\mathbb{C}[[\hbar]]$-algebras $\Phi\colon \widehat{U}_{\exp(\hbar)}(L\mathfrak{g})\xrightarrow{\sim}\widehat{Y}_\hbar(\mathfrak{g})$ (with $\widehat{}$ standing for the appropriate completions). These two completions model the behavior of the algebras in the formal neighborhood of $h=0$. The same construction can be applied to the toroidal setting with $q_i=\exp(\hbar_i)$ for $i=1,2,3$ (see [GTL, T1]). In the current paper, we are interested in the more general relation: $q_1=\omega_{mn}e^{h_1/m}, q_2=e^{h_2/m}, q_3=\omega_{mn}^{-1}e^{h_3/m}$, where $m,n\geq 1$ and ω_{mn} is an mn-th root of 1. Assuming ω_{mn}^m is a primitive n-th root of unity, we construct a homomorphism $\Phi^{\omega_{mn}}_{m,n}$ between the completions of the formal versions of $\mathcal{U}^{(m)}_{q_1,q_2,q_3}$ and $\mathcal{Y}^{(mn)}_{h_1/mn,h_2/mn,h_3/mn}$.

Introduction

Given a simple Lie algebra $\mathfrak{g}$, one can associate to it two interesting Hopf algebras: the quantum loop algebra $U_q(L\mathfrak{g})$ and the Yangian $Y_h(\mathfrak{g})$. Their *classical limits*, corresponding to the limits $q\to 1$ or $h\to 0$, recover the universal enveloping algebras $U(\mathfrak{g}[z,z^{-1}])$ and $U(\mathfrak{g}[w])$, respectively. The representation theories of $U_q(L\mathfrak{g})$ and $Y_h(\mathfrak{g})$ have a lot of common features:

-the descriptions of finite dimensional simple representations involve *Drinfeld polynomials*,

-these algebras act on the equivariant K-theories/cohomologies of Nakajima quiver varieties.

However, there was no explicit justification for that until the recent construction from [GTL] (also cf. [G, Section 5]). In [GTL], the authors construct a $\mathbb{C}[[\hbar]]$-algebra isomorphism

$$\Phi\colon \widehat{U}_{e^\hbar}(L\mathfrak{g})\xrightarrow{\sim}\widehat{Y}_\hbar(\mathfrak{g})$$

of the appropriately completed formal versions of these algebras. Taking the limit $h\to 0$ corresponds to factoring by $(\hbar)$ in the formal setting. The *classical limit* of the above isomorphism is induced by $\varprojlim \mathbb{C}[z,z^{-1}]/(z-1)^r\xrightarrow{\sim}\varprojlim \mathbb{C}[w]/(w)^r\simeq \mathbb{C}[[w]]$ with $z^{\pm 1}\mapsto e^{\pm w}$.

In the current paper, we generalize this construction to the case of the quantum toroidal algebras and the affine Yangians of $\mathfrak{sl}_n$ and $\mathfrak{gl}_1$. To make our notations uniform, we use $\mathcal{U}^{(n)}_{q_1,q_2,q_3}$ to denote the quantum toroidal algebra of $\mathfrak{sl}_n$ (if $n\geq 2$) and of $\mathfrak{gl}_1$ (if $n=1$). This algebra depends on three nonzero parameters q_1,q_2,q_3 such that $q_1q_2q_3=1$. We also use $\mathcal{Y}^{(n)}_{h_1,h_2,h_3}$ to denote the affine Yangian of $\mathfrak{sl}_n$ (if $n\geq 2$) and of $\mathfrak{gl}_1$ (if $n=1$). This algebra depends on three parameters h_1,h_2,h_3 such that $h_1+h_2+h_3=0$. For $n\geq 2$, these algebras were introduced long time ago by [GKV, G][1]. However, the quantum toroidal algebra and the affine Yangian of $\mathfrak{gl}_1$ appeared only recently in the works of different people, see [M, FT1, SV1, MO, SV2, T1].

The main result of this paper, Theorem 3.1, provides a homomorphism

$$\Phi^{\omega_{mn}}_{m,n}\colon \widehat{\mathcal{U}}^{(m),\omega_{mn}}_{\hbar_1,\hbar_2}\longrightarrow \widehat{\mathcal{Y}}^{(mn)}_{\hbar_1,\hbar_2}$$

[1] Actually, we will need to modify slightly their construction in the $n=2$ case.

Download English Version:

https://daneshyari.com/en/article/8966122

Download Persian Version:

https://daneshyari.com/article/8966122

Daneshyari.com