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Stability analysis of a couple-stress fluid saturating a porous medium with temperature and pressure dependent viscosity using a thermal non-equilibrium model

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ABSTRACT

A nonlinear stability threshold for convection in a couple stress fluid saturating a porous medium with temperature and pressure dependent viscosity using a thermal non-equilibrium model is found to be exactly the same as the linear instability boundary. This optimal result is important because it shows that linear theory has completely captured the physics of the onset of convection. The effects of couple stress fluid parameter (F), temperature and pressure dependent viscosity (Γ), interface heat transfer coefficient (H), Darcy–Brinkman number ($\tilde{D}a$) and porosity modified conductivity ratio (γ) on the onset of convection have been investigated. Asymptotic analysis for both small and large values of interface heat transfer coefficient (H) is also presented. An excellent agreement is found between the exact solutions and asymptotic solutions.

1. Introduction

Thermal convection in fluid-saturated porous media has been studied widely due to their widespread engineering applications. Some of these applications are geothermal system, underground spread of pollutants, oil reservoir modeling, thermal insulation, electronic cooling, design of chemical catalytic reactors, medical industries and so on. Vafai [1] presented a comprehensive and cohesive study of heat and mass transfer including the development of practical applications for analysis and design of engineering devices and system involving porous media. In modeling a fluid-saturated porous medium, most of the investigations performed assumed a state of local thermal equilibrium (LTE) between the fluid phase and the solid phase at any point in the medium.

Although in many practical applications, involving high speed flows or large temperature differences between the fluid and solid phases, the assumption of local thermal equilibrium is inadequate. So, it is important to take account of the thermal non-equilibrium effects. The earliest analysis of such non-equilibrium effects were presented by Schumann [2], who considered a one-dimensional semi-infinite bed subject to a step-change in the inlet fluid temperature. As the thermal non-equilibrium theory feature strongly in rapid heat transfer from computer chips via use of porous metal foams and their use in heat pipes it is believe that local thermal non-equilibrium theory will play a major role in future developments.

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The reviews of Kuznetsov [3] give detailed information about the thermal non-equilibrium effects of the fluid flow through a porous packed bed. Baytas and Pop [4] studied free convection in a differentially heated square cavity using a thermal non-equilibrium model. Nield and Bejan [5] state the equations which are generally regarded as modeling unsteady heat transfer in a saturated porous medium where LTE does not apply. Instead of having a single energy equation, which describes the common temperature of the saturated media, two equations are used for fluid and solid phase separately. In the two field model, the energy equations are coupled by interface heat transfer coefficient, which account for the heat lost to or gained from other phase. Rees and Pop [6] and Rees [7] studied free convection boundary layer flow from a uniformly hot vertical surface, but the former adopted a boundary layer approach while the latter solved the full elliptic system of governing equations. The problem of two-dimensional steady mixed convection in a vertical porous layer using a thermal non-equilibrium model is investigated numerically by Saeid [8], while Saeid and Mohamad [9] investigated free convection from a vertical heated plate immersed in porous media driven by sinusoidal plate temperature oscillation. A linear stability analysis and weak nonlinear stability analysis of thermal convection in a rotating porous layer using a thermal non-equilibrium model has been carried by Malashetty et al. [10] and the effect of local thermal non-equilibrium on the onset of convection in a porous medium layer saturated by a Nanofluid is analytically studied by Kuzentsov and Nield [11]. The two equation energy model used for the fluid and solid phase has been found effective for the applications where the fluid and solid phase thermal conductivity is different and when there is heat transport between fluid and solid phase.

Energy method of nonlinear stability theory is based on the study of time evolution of energy of the perturbations to basic flow and lead to a variational problem for a critical dimensionless number, below which the perturbed energy decays to zero. A detailed discussion of the literature pertinent to energy method can be found in Joseph [12–14], Galdi [15], Galdi and Straughan [16], Mulone and Rionero [17], Galdi and Padula [18], Guo et al. [19], Guo and Kaloni [20], Payne and Straughan [21], Kaloni and Qiao [22] and Straughan [23,24]. Straughan [25] has considered a problem of thermal convection in a fluid saturated porous layer using a global non-linear stability analysis with thermal non-equilibrium model. He established that the global nonlinear stability boundary obtained by using local thermal non-equilibrium theory is exactly the same as the linear instability ones found by Banu and Rees [26] and of Malashetty et al. [27]. He has established the equivalence of the linear instability and nonlinear stability boundaries for the thermal convection in a rotating porous layer with the Darcy law using thermal non-equilibrium model. Sunil and Mahajan [28–30] have studied the nonlinear stability analysis for magnetized ferrofluid by using energy method. They found that the nonlinear critical stability magnetic thermal Rayleigh number does not coincide with that of the linear instability analysis, and thus indicates that the subcritical instabilities are possible. More recently, Sunil et al. [31] studied the nonlinear stability analysis for ferroconvection in a porous layer by using a local thermal non-equilibrium model.

With the growing importance of non-Newtonian fluids in modern technology and industry, investigations on such fluids are desirable. Stokes [32] formulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanisms of lubrication of synovial joints, which has become the object of scientific research. According to the theory of Stokes [32], couple-stresses are found to appear in noticeable magnitude in fluids with very large molecules.

At normal operating conditions, the viscosity of an incompressible fluid is assumed to be independent of the pressure. However, it is well known that the viscosity of a fluid can change with pressure, and if the pressure range is significantly large, the viscosity can change by several orders of magnitude. Thus one could consider such liquids as incompressible fluids with pressure dependent viscosities. In his celebrated paper on the response of fluids, Stokes [33] noted that the viscosity of a fluid could depend upon the pressure. However, based on the experiments of Du Buat [34] on the flow of water in canals and normal operating conditions, Stokes suggested that the viscosity could be considered a constant for flows. Stokes is however very careful to delineate the class of flows wherein viscosity might be considered a constant and he also remarks that such an assumption would be invalid under other flow conditions. More recently, Laun [35] has modeled the viscosity of polymer melts through

$$\mu\left(p,T_f\right) = \mu_0 \exp\left[\delta(p-p_0) - \gamma'\left(T_f - T_U\right)\right], \tag{1}$$

where μ_0 is the viscosity at pressure p_0 and temperature T_U; δ and γ' are non-negative constants. There have been numerous other experiments by Bair and co-workers that show that the dependence of the viscosity on the pressure is exponential [see recent experiments of Bair and Kottle [36]). Mention must also be made of the work of Martin-Alfonso and co-workers [37,38] wherein an intricate relationship between the temperature, viscosity and pressure are provided for bitumen.

Rajagopal et al. [39] extended the approximation due to Oberbeck and Boussinesq to the case of a fluid whose viscosity, specific heat and thermal conductivity depend on both the temperature and pressure. They also mentioned that the variation in the viscosity with pressure is far more dramatic than the variation of other quantities with pressure. When the material parameters depend only on the temperature, the result established by Rajagopal et al. [39] reduces to the classical Oberback-Boussinesq approximation. Using this approximation, Rajagopal et al. [40] have studied the problem of Rayleigh–Bènard convection and assuming that the viscosity is an analytic function of the temperature and pressure they studied both the linear as well as the nonlinear stability corresponding to the Rayleigh-Bènard problem. They showed that the principle of exchange of stabilities holds and that the critical Rayleigh numbers for the linear and nonlinear stability coincide. Recently, Rajagopal et al. [41] extended the classical Oberbeck-Boussinesq type of approximation to the case of convection taking place in a porous media and obtained a systematic approximation governing the convection-diffusion that takes place in the porous media. More recently, Sunil et al. [42–44] studied the global stability analysis for thermal convec-