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Zero-sum polymatrix games with link uncertainty: A Dempster-Shafer theory solution

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ABSTRACT

Polymatrix games belong to a class of multi-player games, in which players interact pairwisely and the underlying pairwise interactions are defined by a simple undirected graph where all the edges are completely deterministic. But the link uncertainty between players is not taken into consideration in a standard polymatrix game. In this paper, we put our attention to a special class of polymatrix games — zero-sum polymatrix games, and aim to investigate zero-sum polymatrix games with uncertain links. By considering the diversity of uncertainty, we utilize Dempster-Shafer evidence theory to express the link uncertainty in the games. Then, based on a generalized minmax theorem, we develop a new linear programming model with two groups of constraints to calculate the equilibrium payoffs of players and find the equilibria of the zero-sum plymatrix games with belief links. In terms of these, we also establish a Dempster-Shafer theory solution to zero-sum polymatrix games with link uncertainty. Finally, a numerical example is given to illustrate the potential applications of the proposed model.

1. Introduction

Competition and interaction are ubiquitous in the realistic life. As a special extension of decision theory, game theory has provided a mathematical framework to deal with the interactions of related individuals in interactive decision situations where the aims, goals and preferences of the participating agents are potentially in conflict [1]. Up to now, it has been successfully used in a myriad of disciplines and fields [2–10], including economics, politics, military strategy, management science, etc. For example, over the past decades game theory has obtained successful application in mathematical biology and social science, which, in detail, are named as evolutionary games and population dynamics [11–19]. In terms of the amount of participating players, games can also be classified into two-person games and multi-person games.

In this paper, we focus on a special class of multi-person games, which is called polymatrix games [20]. A polymatrix game [21–25], also known as multimatrix game, is at the basis of an interaction graph. Each node of the underlying graph of the polymatrix games represents a player, each edge connecting every pair of nodes defines a two-person bimatrix game. Given a strategy profile of all the players, in polymatrix games the payoff of a player is the sum of the payoff it obtains via all the pairwise games. Moreover, finding the Nash equilibria of a polymatrix game is quite difficult [26]. For instance, Ref. [27] has proved that it is PPAD-hard to find ϵ-Nash equilibrium of n-person polymatrix games for a constant ϵ. In this

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sense, computing approximate Nash equilibria of polymatrix games attracts much interest [24,28–30]. Some recent works have developed a linear programming approach to efficiently derive Nash equilibria of zero-sum polymatrix games [31,32]. This approach fast achieves a multiplayer generalization of von Neumann's minmax theorem [33], which is originally only applicable to two-person zero-sum games.

As presented above, a standard polymatrix game is formally defined on a simple undirected graph, where all the edges are completely deterministic, so that any pair of players is either linked or unlinked and the link uncertainty between players is not taken into consideration. However, the realistic situations are often inconsistent with this hypothesis. In reality, a player may have different probabilities to play games with other players. In the paper, our objective and motivation is to study the polymatrix games with such link uncertainty. Because of the difficulty of finding Nash equilibria to a general polymatrix game, our attention will be given only to the zero-sum polymatrix games with link uncertainty. By considering the diversity of uncertainty, such as probabilistic, linguistic, and intervallic, we utilize Dempster-Shafer evidence theory [34,35], an effective tool of modeling and reasoning in uncertain environment, to express the link uncertainty in ploymatrix games. Moreover, the generalized minmax theorem [31,32], frequently used in the zero-sum plymatrix games, is effectively extended to the environment with link uncertainty. Based on these, we have developed a Dempster-Shafer theory solution to the zero-sum polymatrix games with link uncertainty.

To sum up, the contributions of this paper mainly include three aspects: (i) The uncertainty is imported to the polymatrix game by considering the link uncertainty of the underlying graph of a polymatrix games, which, to some extent, extends the original deterministic game model. (ii) By taking note of the diversity of the types of uncertainties, Dempster-Shafer evidence theory is employed to express various kinds of uncertainties, which further provides a unified framework of uncertainty expression for ploymatrix games with link uncertainty. iii) Based on the generalized minmax theorem, an approach is developed to obtain the feasible region of equilibria for the zero-sum ploymatrix games with belief links, which essentially transfers the uncertainty in links to the payoffs. Compared with the work presented in [31,32], our this study extends the conventional zero-sum polymatrix games to uncertain environment with link uncertainty, which gives a more generalized model. Specially, since the developed approach to find the equilibria of the extended game model is based on the generalized minmax theorem given in [31,32], it has a desirable feature that the proposed approach can be totally reduced to the linear programming method given in [31,32] if the link uncertainty is removed.

The paper is organized as follows. Section 2 introduces the classical zero-sum polymatrix games. In Section 3, the zero-sum polymatrix games with link uncertainty are presented and we show the Dempster-Shafer theory solution for this kind of games. After that, an illustrative example is given in Section 4. Finally, Section 5 concludes this paper.

2. Zero-sum polymatrix games

2.1. Polymatrix games

Polymatrix Games are a special class of multi-player games, in which the interactions among players are pairwise. Therefore, a polymatrix game is usually expressed in the form of graphes. For a graph $G = (V, E)$ where V is the set of nodes and E represents the set of edges, the players are associated with the nodes of G, and the existing edges in E correspond to bimatrix games. Formally, a polymatrix Game is defined as follows.

Definition 1. Let $G = (V, E)$ be an undirected simple graph, where $V = \{1, \ldots, n\}$ is the set of nodes and E is the set of all the edges in G. For $i, j = 1, \ldots, n,$ each player i has a finite pure strategy set S_i, and each edge $e_{ij} \in E$ defines a two-person game (A_{ij}, A_{ji}), where $A_{ij} \in \mathbb{R}^{|S_i|\times|S_j|}$ is the payoff matrix of player i against j, and similarly $A_{ji} \in \mathbb{R}^{|S_j|\times|S_i|}$. The polymatrix game $\mathcal{G} = (G, (A_{ij}, A_{ji})_{(i,j)\in E})$ is a n-person game with the payoff function $\pi_i : \Delta_1 \times \cdots \times \Delta_n \mapsto \mathbb{R}$:

$$\pi_i(\mathbf{s_1}, \ldots, \mathbf{s_n}) = \sum_{j\in N(i)} \mathbf{s_i}^T A_{ij}\mathbf{s_j}, \quad i \in V, \;\; \mathbf{s_i} \in \Delta_i \tag{1}$$

where $N(i)$ is the neighbourhood of player i, and $\mathbf{s_j}$ is the strategy vector of player j, $j \in V$.

According to the above definition, in a polymatrix game each player chooses a single strategy for all his bimatrix games and receives the sum of the payoffs from his bimatrix games.

2.2. Zero-sum Polymatrix games and their equilibria

Since it is very hard toderive Nash equilibria of general polymatrix games, some researchers have put their attention to zero-sum polymatrix games, and obtained very important progress. Let us first introduce the definition of zero-sum polymatrix games and then give the approach of finding Nash equilibria for such a class of polymatrix games.

Basically, there are two ways to establish a zero-sum polymatrix game. One is to directly define all edges in a polymatrix game are zero-sum, as shown in [31,36]. The other is more general through making zero-sum being the global feature of the game [32]. The formal expression of these two means are given as below.

Definition 2. A polymatrix game $\mathcal{G} = (G, (A_{ij}, A_{ji})_{(i,j)\in E})$ is zero-sum if for all $e_{ij} \equiv (i, j) \in E$,

$$A_{ij} = -A_{ji}^T. \tag{2}$$