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Modelling a dynamic market potential: A class of automata networks for diffusion of innovations

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ABSTRACT

Innovation diffusion processes are generally described at aggregate level with models like the Bass Model (BM) and the Generalized Bass Model (GBM). However, the recognized importance of communication channels between agents has recently suggested the use of agent-based models, like Cellular Automata. We argue that an adoption or purchase process is nested in a communication network that evolves dynamically and indirectly generates a latent non-constant market potential affecting the adoption phase.
Using Cellular Automata we propose a two-stage model of an innovation diffusion process. First we describe a communication network, an Automata Network, necessary for the "awareness" of an innovation. Then, we model a nested process depicting the proper purchase dynamics. Through a *mean field approximation* we propose a continuous representation of the discrete time equations derived by our nested two-stage model. This constitutes a special non-autonomous Riccati equation, not yet described in well-known international catalogues. The main results refer to the closed form solution that includes a general dynamic market potential and to the corresponding statistical analysis for identification and inference. We discuss an application to the diffusion of a new pharmaceutical drug.

1. Introduction

Since the publication of the Bass model in 1969, research on diffusion of innovations and innovation theory have raised a growing interest, with reference both to consumers behaviour (see Gatignon and Robertson [1]) and marketing management for developing new strategies focused on potential adopters or potential adoption units. Interesting reviews of the literature on diffusion models are provided by Mahajan and Muller [2]), Mahajan *et al.* [3], Mahajan *et al.* [4], Meade and Islam [5] and Muller *et al.* [6] where it is highlighted that the purpose of the diffusion model is to describe the successive increases in the number of adoptions or purchases and predict the continued development of a diffusion process already in progress. In spite of the more recent research proliferation in this field, the basic and fundamental known diffusion models are those of Fourt and Woodlock [7], Mansfield [8] and Bass [9]. The last one (BM) is an extension of the other two and assumes that potential adopters (or adoption units) are influenced in their purchase behaviour by two sources of information: an external, like mass-media communication and an internal, word-of-mouth. Furthermore, it is assumed that adopters can be influenced only by one of these two forces, forming two distinct groups, innovators (mass-media) and imitators (word-of-mouth) and therefore, part of adoptions is based on learning by imitation and part of them does not. Formally, the model can be expressed through a first order differential equation

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t)). \tag{1}$$

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Instantaneous purchases, $z'(t)$, at time t are proportional to the residual market $(m-z(t))$, where m is the assumed fixed market potential, and determined by two additive components. The first one, $p(m-z(t))$, refers to innovators, who adopt with a rate p called coefficient of innovation. The group of innovators is essential for the "take-off'" of diffusion, even if present at any stage of the process.

The second part of Eq. (1), $q(z(t)/m)(m-z(t))$, represents purchases of buyers at time t who are influenced by previous realized adoptions (word-of-mouth effect, w-o-m for short) through parameter q. The effect of parameter q is modulated by the ratio $z(t)/m$, justifying the temporal delay of purchases due to w-o-m effect. In what follows we assume that w-o-m describes both direct interpersonal communication including "auto-communication" (learning and memory effects) in different times and signals due to realized adoptions.

If innovators are necessary for the initial phase of the diffusion process, imitators are crucial for its development and growth, the life cycle of an innovation depending on these two combined effects.

An equivalent interpretation of Eq. (1) is based on the dual hazard rate specification, i.e., $h(t)=z'(t)/(m-z(t))=p\cdot 1+q\cdot z(t)/m+(1-p-q)\cdot 0$, where p,q and $1-p-q$ are group conditioning probabilities with $1-p-q$ pertaining to "neutrals" subgroup and 1, $z(t)/m$ and 0 are the corresponding conditional probabilities towards adoption. Summation over previous joint probabilities describes the usual marginalization technique based on the law of total probability.

An extremely useful extension of the Bass model is represented by the Generalized Bass Model (GBM) by Bass *et al.* [10] allowing to include the presence of exogenous interventions (strategic interventions, policies, marketing strategies). The GBM equation is

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))x(t) \tag{2}$$

where $x(t)$ denotes a quite general intervention function with neutral level 1, whose effect can accelerate or delay purchases over time, i.e., control the geometry of time as we can see by inspecting, for instance, the closed form solution of Eq. (2) in [10]. Function $x(t)$ cannot control the market potential m or the intrinsic diffusion parameters p and q.

One of the main assumptions in the Bass models relates to the market potential (or carrying capacity), m, whose size is considered fixed along the whole diffusion process. However, it is a common experience that such a potential may have a variable structure during the product life cycle, due to endogenous or exogenous factors.

The issue of a dynamic market potential is not new to the diffusion literature. We may separate two different formal approaches: Some papers introduce only a modification of the residual market, $(m-z(t))$ and exclude an intervention on the w-o-m ratio. See, for instance, Mahajan and Peterson [11], Horsky [12], Kamakura and Balasubramanian [13] and Mesak and Darat [14].

A second group allows for both modifications. Among others, we consider Sharif and Ramanathan [15], Jain and Rao [16], Parker [17,18], Rao [19], Kim *et al.* [20], Goldenberg *et al.* [21] and Centrone *et al.* [22].

From a mathematical point of view there are different assumptions governing the shape of $m(t)$. In some cases it is exogenously determined as a function of observed variables, e.g., in Mahajan and Peterson [11], Kalish [23], Jain and Rao [16], Parker [17,18], Horsky [12], Kim *et al.* [20] and Kamakura and Balasubramanian [13]. Some effort is necessary for a correct specification of the main drivers (population, prices, number of households with special facilities, number of competitors, number of retailers, threshold probabilities, etc.) and a suitable transformation to attain a reasonable correspondence with the scale of the adoption process.

In few cases the market potential is assumed to follow a simple exponential function of time, Sharif and Ramanathan [15], Centrone *et al.* [22] and Meyer and Ausubel [24].

In this paper we propose a model in which the market potential is a function of time, $m(t)$, and may assume various levels during the product life cycle. We observe that such a variability is particularly evident in the first part of diffusion, the so called *incubation period*, when the success of an innovation is still uncertain and may depend on several elements, eventually depressing sales. We argue that in this phase marketing and management activities play a crucial role in stimulating the product "take off".

However, there are situations in which these efforts are not sufficient for overcoming the initial crisis: a typical example is represented by those goods that produce network externalities effects due to technological constraints (mobile phones, fax-machines, etc.). Thus we focus our research on goods that do not exhibit network externalities effects. We concentrate our interest on "stand alone goods" whose adoption may be highly facilitated by promotional activities and, in particular, we examine a new pharmaceutical drug's diffusion.

As is well-known, advertising and other forms of product promotion exert a major effect in the launch phase, while other kinds of communication like word-of-mouth may better explain later purchases.

We use this consideration for motivating the time dependence of the market potential and we adopt an evolutionary perspective for providing a theoretical explanation of this intuition. In particular, we develop a model in which communication and adoption processes are separate but co-evolutionary phases in diffusion.

The main idea expressed in our model may find a precursory version in Sawhney and Eliashberg [25]. The Authors propose a two-phase approach by separating two notions of time: the *time to decide*, T, and the *time to act*, τ. The time to adopt is the sum of the previous two, $t=T+\tau$, and its distribution is a convolution under very simple exponential frameworks. However, our approach is different, being based on the direct product of two independent distributions that define the normalized market potential (awareness-persuasion) and the corresponding normalized purchase process.