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On *S*-curves and tipping points

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Abstract

In his discussion in this journal of Kurzweil's *The Singularity is Near*, Modis criticizes Kurzweil's loose characterization of the "knee" of a growth curve. Likewise, the "tipping points" described by Gladwell (*The Tipping Point*) are clearly relevant to forecasting systems, but Gladwell did not mathematically identify such points. Both concepts refer to a point on the curve where growth accelerates dramatically and sustains itself. What can be said in a rigorous way about knees and tipping points in growth systems?

The answer has to do with the number of parameters of the growth curve, and the (probabilistic) model underlying the growth curve. Using probability theory and computational experiments, this paper clarifies these points for the logistic and Bass curves (identifying an unambiguous tipping point for the latter), and explores the merits of a 3-parameter model of innovation adoption. It concludes that if forecasters are to deal scientifically with the now-established management notion of "tipping points," a 3-parameter model is needed. The paper also resolves four minor but annoying paradoxes in the growth curve literature.

Keywords: Technology forecasting; *S*-curves; Tipping point; Logistic growth; Bass model; Innovation adoption; Homeostasis; Change management

1. Introduction

In his discussion of Kurzweil's *The Singularity is Near* [1], Modis [2] criticizes Kurzweil's loose characterization of the "knee" of a growth curve. Gladwell's [3] "tipping points," though clearly relevant to the kinds of systems that interest readers of this journal, similarly remain formally undefined. "Knee"

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and “tipping point” both refer to a point on the curve where growth accelerates dramatically and sustains itself.

Discussion of these matters in business and popular literature can be vague or misleading. Neither Kurzweil nor Gladwell provided a mathematical definition for a tipping point or knee. Kurzweil’s use of “singularity” to denote a coincidence of the knees of many growth curves is jarring to mathematicians to whom the word means something entirely different. Calculus students, who know the inflection point in an *S*-curve is where things start to slow down, would have been confused by Andy Grove’s [4] use of “strategic inflection point” to mean “the nightmare moment… when massive change occurs and a company must, virtually overnight, adapt or fall by the wayside.” As *Technological Forecasting and Social Change* Senior Editor Hal Linstone has emphasized repeatedly, these moments happen at the tails of *S*-curves, not in their middles.

Though not supplemented by formal mathematical modeling, Gladwell’s sociological observations are meticulous and strongly suggest the reality of tipping points of some sort, in a variety of situations. That he is now one of the most sought-after speakers in the United States indicates further that his observations resonate with those of managers and forecasters, who will be putting the “tipping point” idea to use, with or without scientific guidance.

What can be said in a rigorous way about tipping points in growth systems? The answer has to do with the number of parameters of a growth curve, and the (probabilistic) model underlying that growth curve. Using probability theory and computational experiments, this paper clarifies these points for the logistic (one- and two-parameter) and Bass (2-parameter) curves, and also explores the merits of a dynamic 3-parameter model of innovation adoption, concluding that it, or other 3-parameter models, are needed if forecasters are to deal scientifically with the now-established management notion of “tipping points.”

The paper also resolves four minor but annoying paradoxes having to do with positive external influence on adoption, imitation effects, and active resistance to change.

2. Exponential and logistic curves

2.1. Exponential growth

The differential equation for exponential growth [5] is

$$\mathrm{d}x/\mathrm{d}t = sx \tag{1}$$

At any moment, the incremental growth is proportional to the existing population size. In propagating species, this means births are proportional to the size of the reproducing population. Applied to innovation adoption situations, Eq. (1) implies a pure imitation effect, with incremental adoption proportional to the number of earlier adopters.[1]

[1] In this discussion of innovation adoption, “external effects” refer to influences originating outside the adopting and potentially adopting population. These influences are usually purposeful, and include advertising and (for intra-organizational innovations) exhortations from management urging adoption among the rank and file. “Internal effects” or internal influences, on the other hand, originate within the (potentially) adopting population. They include any reason one adopter may have for wishing to follow the example of an earlier adopter, and are motivated by the desires to imitate one’s neighbors or colleagues, to “keep up with the Joneses,” and so on. This terminology was introduced by Bass [14].