Cognitive Development 30 (2014) 30–46

Contents lists available at ScienceDirect

Cognitive Development

Organization matters: Mental organization of addition knowledge relates to understanding math equivalence in symbolic form☆

Dana L. Chesney[a,*], Nicole M. McNeil[a,**], Percival G. Matthews[a,1], Caroline E. Byrd[a], Lori A. Petersen[a,2], Mary C. Wheeler[a], Emily R. Fyfe[b], April E. Dunwiddie[a]

[a] Department of Psychology, University of Notre Dame, United States
[b] Department of Psychology and Human Development, Vanderbilt University, United States

ARTICLE INFO

Keywords:
Conceptual development
Mental organization
Mathematical understanding
Mathematical knowledge
Individual differences

ABSTRACT

Two quasi-experiments examined mental organization of addition knowledge as a potential source of individual differences in understanding math equivalence in symbolic form. We hypothesized that children who mentally organize addition knowledge around conceptually related groupings would have better understanding of math equivalence. In Quasi-experiment 1, we assessed 101 second and third grade students' mental organization of addition knowledge based on their use of decomposition strategies to solve addition problems (e.g., $3+4=3+3+1=6+1=7$). In Quasi-experiment 2, we assessed 94 second grade students' mental organization based on their ability to generate a set of equations equal to a target value. In both quasi-experiments, children whose mental organization better reflected conceptually related groupings exhibited better understanding of math equivalence. Results thus support the

☆ This research was supported by grant R305B070297 from the U.S. Dept. of Education, Institute of Education Sciences. The opinions expressed are those of the authors and do not represent the views of the Institute or the U.S. Dept. of Education. Thanks to Martha Alibali and Heather Brletic-Shipley for their input and Krysten Williams and Jennifer Heil for their help with coding. This research would not have been possible without the support of many administrators, teachers, parents, and students in the greater South Bend, IN and Washington, DC areas.

* Corresponding author. Present address: Department of Psychology, The Ohio State University, 225 Psychology Building, 1835 Neil Avenue, Columbus, OH, 43210, United States. Tel.: +1 862 432 6539.
** Corresponding author at: Department of Psychology, University of Notre Dame, 118 Haggar Hall, Notre Dame, IN, 46556, United States. Tel.: +1 574 631 5678.
E-mail addresses: dlchesney@gmail.com (D.L. Chesney), nmcneil@nd.edu (N.M. McNeil).
[1] Present address: University of Wisconsin-Madison, United States.
[2] Present address: University of Chicago, United States.

http://dx.doi.org/10.1016/j.cogdev.2014.01.001

hypothesis that mental organization of addition knowledge into conceptually related groupings based on equivalent values may influence understanding of math equivalence in symbolic form.

1. Introduction

Although many developmental psychologists seek to identify commonalities in children's development, studying individual differences (Cronbach, 1957; Underwood, 1975) may provide insight into mechanisms of typical development (Hughes et al., 2005; Nelson, 1981) and inspire interventions that facilitate learning in reading (Blachman, Tangel, Ball, Black, & McGraw, 1999; Shaywitz et al., 2004) and mathematics (Booth & Siegler, 2006; Ramani & Siegler, 2008). Here, we examine a source of individual differences in children's understanding of math equivalence.

1.1. Mathematical equivalence

Mathematical equivalence, commonly symbolized by the equal sign (=), is the relation between two interchangeable quantities (Kieran, 1981). Understanding math equivalence in symbolic form not only involves understanding the meaning of the equal sign, but also encoding math equations in their entirety, correctly identifying an equation's two "sides," and noticing relations within equations (Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Rittle-Johnson & Alibali, 1999). To be concise, we herein refer to this array of knowledge as "understanding of math equivalence", although we are specifically referring to understanding of math equivalence in symbolic form.

Understanding of math equivalence is critical to development of algebraic thinking (Falkner, Levi, & Carpenter, 1999; Kieran, 1992; Knuth, Stephens, McNeil, & Alibali, 2006). Unfortunately, most U.S. children have poor understanding of math equivalence (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Falkner et al., 1999; McNeil, 2008; Perry, 1991). McNeil (2005) found that nearly 80% of U.S. 7–11-year-olds solve math equivalence problems—problems with operations on both sides of the equal sign (e.g., $6 + 3 = 4 + \_$)—incorrectly.

1.2. Early learning of arithmetic as a source of difficulty

A growing body of work suggests that difficulties in understanding math equivalence may be largely attributable to children's early learning experiences in mathematics (Baroody & Ginsburg, 1983; Li, Ding, Capraro, & Capraro, 2008; McNeil, 2008; McNeil, Fyfe, Petersen, Dunwiddie, & Brletic-Shipley, 2011). In the U.S., arithmetic problems are almost always presented in an "operations equals answer" format (e.g., $3 + 4 = 7$), which may fail to highlight the interchangeable nature of the two sides (McNeil et al., 2011; Seo & Ginsburg, 2003; but see Wynroth, 1975, as cited in Baroody & Ginsburg, 1983, for an atypical curriculum emphasizing relational meanings). As a result, many children come to interpret the equal sign *operationally*, as a signal to "give the answer," rather than *relationally*, as a signal that both sides share a common value (Baroody & Ginsburg, 1983; Behr et al., 1980; McNeil & Alibali, 2005a). Although operational interpretations of the equal sign are valid in some contexts (Seo & Ginsburg, 2003), they are inappropriate and often detrimental in algebraic contexts (Knuth et al., 2006; McNeil, Rittle-Johnson, Hattikudur, & Petersen, 2010). Consequently, most U.S. elementary school children not only fail to solve math equivalence problems correctly, but also fail to encode such problems' features correctly (McNeil & Alibali, 2004).

However, not all U.S. children exhibit such errors. What is it about the 10–25% of children who demonstrate understanding of math equivalence that enables them to extract appropriate patterns from their formal experiences with arithmetic? General competence or math ability alone cannot explain individual differences in understanding of math equivalence. Computational fluency, grade level, and age have not been consistently correlated with understanding of math equivalence in 7- to 11-year-olds. Some studies report no associations (Carpenter, Levi, & Farnsworth, 2000; McNeil &